Distributed Temperature Regulator Design for an Air Conditioning System

Takumi Ido * **Takayuki Ishizaki** * **Jun-ichi Imura** *
Yuki Katsuyama ** **Masahiko Murai** ** **Katsuya Yokokawa** **

* *Graduate School of Engineering, Tokyo Institute of Technology; 2-12-1, Ookayama, Meguro, Tokyo, 152-8552, Japan. e-mail: {ido,ishizaki,imura}@cyb.mei.titech.ac.jp.*
** *Power and Industrial Systems R&D Center, Toshiba Corporation; 1, Toshiba-Cho, Fuchu-Shi, Tokyo, 235-8523, Japan. e-mail: {yuki1.katsuyama,masahiko.murai,katsuya.yokokawa}@toshiba.co.jp.*

Abstract: In this paper, we propose a design method of distributed temperature regulators for an air conditioning system, which is modeled as a network system composed of second-order subsystems representing the dynamics of air conditioners and thermal diffusion among areas. In the proposed method, to make a distributed regulator design problem tractable, we consider confining the class of distributed controllers to that consisting of second-order subcontrollers, whose network structure is made identical to that of the controlled system. On the basis of this specialization, we derive a necessary and sufficient condition of stabilizing controller parameters. Furthermore, we show that the problem of distributed regulator design can be reduced to a problem of robust state feedback gain design, formulated as guaranteeing a control specification in terms of the H_2- and H_∞-norm. The efficiency of our distributed regulator design method is shown through a couple of numerical simulations.

Keywords: Distributed control, Output regulation, Disturbance rejection, Air conditioning systems, H_2/H_∞-control specifications.

1. INTRODUCTION

The design of smart buildings [Tadokoro et al. (2014)] is gaining attention with recent developments in computing and networking technologies. In smart buildings, a variety of mechanical and cyber devices, sophisticated control systems, and so forth are integrated to realize an energy efficient housing and working environment. In particular, the HVAC (heating, ventilating, and air conditioning) system design becomes a key component to construct such a smart system [Anderson et al. (2008)]. In fact, a number of works related to HVAC system design can be found in the literature, e.g., [Elliott and Rasmussen (2013); Mondal and Bhattacharya (2014)], which are based on the premise of decentralized control.

With this background, this paper is concerned with the design of distributed temperature regulators for an air conditioning system. More specifically, modeling the dynamics of air conditioners and thermal diffusion among several areas as a network system composed of second-order subsystems, we consider developing a distributed regulator for a set of air conditioners such that the temperature of each area is steered towards a reference temperature, under the existence of unknown constant heat loads. In general, distributed (or decentralized) control has an advantage to substantially reduce the communication costs among subcontrollers, because each subcontroller communicates only with their neighboring controllers while centralized control requires the information of all subcontrollers. However, it should be noted that the design problem of such a structured control system is not necessarily easy to solve, as discussed in the literature; see [Blondel and Tsitsiklis (2000); Papadimitriou and Tsitsiklis (1986)] for a discussion on computational intractability and [Rotkowitz and Lall (2006)] for a characterization of convex problems, called quadratic invariance, in a decentralized control formulation.

To overcome this difficulty, we consider confining the class of distributed controllers to that consisting of second-order subcontrollers. The network structure among subcontrollers, whose communication is implemented as a type of flocking algorithms [Olfati-Saber (2006)], is made identical to that of the network system to be controlled. Owing to this specialization, it turns out that a necessary and sufficient condition of stabilizing controller parameters is obtained. Furthermore, the problem of distributed controller design can be reduced to a problem of robust state feedback gain design, which is to be formulated to guarantee a control specification in terms of the H_2- and H_∞-norm.

To make our contribution clearer, we provide several references on distributed controller design for network systems. In [Andreasson et al. (2014a)], a class of network systems is considered on the basis of a distributed PI control formulation, where a type of consensus algorithms [Olfati-Saber and Murray (2004)] is implemented. As an extension of this work, [Andreasson et al. (2014b)] is concerned with an output regulator design problem. In these works, sev-

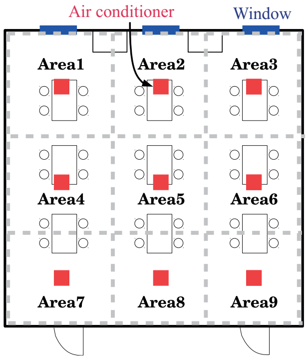

Fig. 1. Air conditioning system for office room.

eral conditions on stabilization and output regulation are derived for network systems with unknown disturbances. However, since they focus on a specific class of network systems, the stability conditions for the distributed PI controllers cannot straightforwardly be applied to our air conditioning system, due to the difference of available input ports.

An approach to convexify a distributed controller design problem has been proposed in [Langbort et al. (2004)], which derives a sufficient condition in a form of linear matrix inequalities. Even though this method can produce a distributed output feedback controller having the same interconnection structure as network systems to be controlled, the network systems are required to comply with a particular dissipative condition, which is possibly restrictive in practice. Furthermore, by focusing on spatially invariant systems, distributed controller design methods in terms of quadratic criteria have been developed in the formulations of both continuous and discrete spaces [Bamieh et al. (2002); D'Andrea and Dullerud (2003)].

The remainder of this paper is organized as follows. In Section 2, we first formulate a distributed temperature regulator design problem for the air conditioning system. Next, in Section 3, giving a specific form of distributed controllers, we analyze the stability as well as control performance of the closed-loop system. In Section 4, through a numerical simulation on the distributed control of the air conditioning system, we show the efficiency of our distributed control system design method. Finally, concluding remarks are provided in Section 5.

Notation. We denote the identity matrix by I, the set of eigenvalues of A by $\mathrm{spec}(A)$, the Kronecker product of A and B by $A \otimes B$, and the block diagonal matrix having matrices M_i for $i \in \{1, \ldots, n\}$ on its diagonal blocks by

$$\mathrm{diag}(M_1 \ldots, M_n) = \mathrm{diag}(M_i)_{i\in\{1,\ldots,n\}}.$$

The H_∞-norm of a stable transfer matrix and the H_2-norm of a stable proper transfer matrix are defined by

$$\begin{aligned} \|G(s)\|_\infty &:= \sup_{\omega\in\mathbb{R}} \|G(j\omega)\| \\ \|G(s)\|_2 &:= \sqrt{\tfrac{1}{2\pi}\int_{-\infty}^{\infty} \|G(j\omega)\|_{\mathrm{F}}^2 d\omega}, \end{aligned}$$

where $\|\cdot\|$ and $\|\cdot\|_{\mathrm{F}}$ denote the induced 2-norm and the Frobenius norm, respectively.

2. PROBLEM FORMULATION

We consider an air conditioning system for an office room depicted in Fig. 1, which is divided into several areas according to the position of each air conditioner placed in advance. Regarding the dynamics of air conditioners and thermal diffusion among the areas as a network system whose graph is undirected and connected, we model the dynamics of the ith subsystem, i.e., the ith area, as

$$\begin{cases} \tau_i \dot{Q}_i &= -Q_i + u_i \\ c_i \dot{T}_i &= -\sum_{j\in\mathcal{N}_i} k_{i,j}(T_i - T_j) + Q_i + l_i \end{cases} \quad (1)$$

where the first equation represents the dynamics of the air conditioner and the second represents the thermal diffusion among the areas. In particular, the state variables Q_i and T_i denote the heat quantity from the ith air conditioner and the temperature of the corresponding area, and the external input signal u_i represents a command to regulate the heat quantity. For the positive constants, τ_i denotes the time constant of the ith air conditioner, c_i denotes the thermal capacity of the ith area, and $k_{i,j} = k_{j,i}$ denotes the thermal conductance between the ith and jth areas, for which $\mathcal{N}_i$ denotes the index sets associated with the neighborhood of the ith area. Furthermore, l_i denotes the thermal load placed at the ith area, which can be regarded as an unknown constant disturbance.

For convenience of notation, we rewrite (1) as

$$\Sigma_i : \begin{cases} \dot{q}_i &= -\alpha_i q_i + u_i \\ \dot{x}_i &= -\beta_i \sum_{j\in\mathcal{N}_i} \gamma_{i,j}(x_i - x_j) + q_i + d_i, \end{cases} \quad (2)$$

where q_i and x_i are the state variables, u_i is the external input signal, d_i is a constant disturbance, and α_i, β_i, and $\gamma_{i,j} = \gamma_{j,i}$ are positive constants. In the following, assuming that the subsystem states are measurable only locally, we consider designing a distributed control system in which each of subcontrollers is allowed to communicate with its neighboring subcontrollers. In particular, each of subcontrollers is supposed to be in the form of

$$C_i : u_i = \mathcal{K}_i(q_i, x_i, \{\xi_j\}_{j\in\mathcal{N}_i}; r_i), \quad (3)$$

which implies that the ith subcontroller, whose dynamical map is denoted by $\mathcal{K}_i(\cdot)$, uses the information of the local states q_i and x_i, the communication signals ξ_j from its neighboring subcontrollers indexed by $j \in \mathcal{N}_i$, and a reference signal r_i. In this formulation, we aim at finding a set of subcontrollers, which form a distributed controller, such that

$$\lim_{t\to\infty} x_i - r_i = 0, \quad \forall i \in \mathcal{N} \quad (4)$$

for any unknown constant disturbance d_i, where $\mathcal{N}$ denotes the index set associated with the subsystems. A schematic depiction of the distributed control system is shown in Fig. 2.

3. DISTRIBUTED CONTROLLER DESIGN

3.1 Formulation of Distributed Controller

In Section 2, we have formulated a distributed controller design problem for a class of network systems, composed of the second-order subsystems Σ_i in (2). In general, the design problem of such a structured control system is not necessarily easy to solve, as discussed in the literature; see, e.g., [Blondel and Tsitsiklis (2000); Papadimitriou

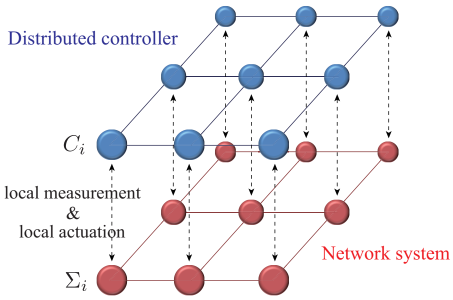

Fig. 2. Schematic depiction of distributed control systems.

and Tsitsiklis (1986)]. Thus, to make our distributed controller design problem tractable, we consider confining the class of distributed controllers to that consisting of second-order subcontrollers. Owing to this specialization, it will turn out that a necessary and sufficient condition of stabilizing controller parameters is obtained. Furthermore, the problem of distributed controller design can be reduced to a problem of robust state feedback gain design.

In the following, the subcontroller dynamics in (3) is supposed to be given as

$$C_i : \begin{cases} \dot{\eta}_i &= \xi_i \\ \dot{\xi}_i &= (x_i - r_i) - \beta_i \sum_{j\in\mathcal{N}_i} \gamma_{i,j}(\xi_i - \xi_j) \\ u_i &= (\boldsymbol{K}_0 + \alpha_i)q_i + \boldsymbol{K}_1(x_i - r_i) + \boldsymbol{K}_2\xi_i + \boldsymbol{K}_3\eta_i \end{cases} \tag{5}$$

where $\boldsymbol{K}_j$ for $j \in \{0, 1, 2, 3\}$ are the controller parameters to be designed. This distributed controller has the following features:

- The integral action of η_i in (5) works as achieving the regulation of (4) for unknown constant disturbances d_i. This can be seen from the fact that, when the distributed control system stays at an equilibrium state, i.e., $\dot{\eta}_i = 0$ and $\dot{\xi}_i = 0$, it follows that
$$\xi_i^* = 0, \quad x_i^* = r_i, \quad \forall i \in \mathcal{N},$$
where ξ_i^* and x_i^* denote the equilibria of the corresponding variables. In addition, the equilibrium of the integrator η_i^* is to be determined such that
$$\boldsymbol{K}_0 q_i^* + \boldsymbol{K}_3\eta_i^* = 0 \quad \forall i \in \mathcal{N},$$
where q_i^* is determined as complying with d_i in (2).
- The network structure of the subcontrollers K_i is made identical to that of Σ_i; see the dynamics of x_i in (2) and that of ξ_i in (5).
- The local state feedback action of q_i in (5) makes the time constant of the dynamics of q_i in (2) uniform for all $i \in \mathcal{N}$.

Owing to the first feature, in order to attain the specification of (4), it is sufficient to select a set of controller parameters $\boldsymbol{K}_j$ for $j \in \{0, 1, 2, 3\}$, which are not dependent on $i \in \mathcal{N}$, such that the distributed control system is stable. Furthermore, the second and third features allow us to represent the distributed control system as the matrix form of

$$\begin{bmatrix} \dot{\eta} \\ \dot{\xi} \\ \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & -D\Gamma & I & 0 \\ 0 & 0 & -D\Gamma & I \\ \boldsymbol{K}_3 I & \boldsymbol{K}_2 I & \boldsymbol{K}_1 I & \boldsymbol{K}_0 I \end{bmatrix} \begin{bmatrix} \eta \\ \xi \\ x \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ -r \\ d \\ -\boldsymbol{K}_1 r \end{bmatrix} \tag{6}$$

where the symbols without the subscript i denote their stacked versions, D denotes the diagonal matrix whose diagonal elements are β_i, and Γ denotes the symmetric weighted graph Laplacian matrix whose (i, j)-element is given as

$$\Gamma_{i,j} = \begin{cases} \sum_{j\in\mathcal{N}_i} \gamma_{i,j}, & j = i \\ -\gamma_{i,j}, & j \in \mathcal{N}_i \\ 0, & \text{otherwise.} \end{cases}$$

The following arguments are valid not only for the symmetric graph Laplacian of the lattice network as in Fig. 1 but also for that of any network structure.

3.2 Stability Analysis and Control Performance Analysis

To simplify notation, let us equivalently rewrite the distributed control system in (6). First, we introduce a diagonal scaling of the state variables in (6). More specifically, scaling each of the state variables by $D^{-\frac{1}{2}}$, e.g., replacing x with $D^{-\frac{1}{2}}x$, we can represent a normalized version of the state transition matrix of (6) as

$$\widehat{\boldsymbol{A}} := \boldsymbol{A} \otimes I - E \otimes L \tag{7}$$

where $L := D^{\frac{1}{2}}\Gamma D^{\frac{1}{2}}$, which is symmetric positive semidefinite, and

$$\boldsymbol{A} := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \boldsymbol{K}_3 & \boldsymbol{K}_2 & \boldsymbol{K}_1 & \boldsymbol{K}_0 \end{bmatrix}, \quad E := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8}$$

Note that L has the same network structure, i.e., the Boolean structure, as that of Γ, and it possesses at least one zero eigenvalue. Next, we represent the distributed control system as a deviation system. Considering the deviation from an equilibrium for a constant disturbance d, we obtain the deviation system

$$\begin{cases} \dot{\delta} &= \widehat{\boldsymbol{A}}\delta + (R \otimes I)w \\ z &= (S \otimes I)\delta, \end{cases} \tag{9}$$

where $\widehat{\boldsymbol{A}}$ is defined as in (7), and the disturbance input port R and the evaluation output port S are to be selected as complying with a control objective.

On the basis of this representation, the following fact is shown by taking a decomposition approach similar to that in [Hara et al. (2014)].

Lemma 1. For $\widehat{\boldsymbol{A}}$ in (7), it follows that

$$\mathrm{spec}(\widehat{\boldsymbol{A}}) = \bigcup_{\lambda\in\mathrm{spec}(L)} \mathrm{spec}(A_\lambda + B\boldsymbol{K}) \tag{10}$$

where A_λ, B, and $\boldsymbol{K}$ are defined as

$$A_\lambda := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{K} := \begin{bmatrix} \boldsymbol{K}_3 \\ \boldsymbol{K}_2 \\ \boldsymbol{K}_1 \\ \boldsymbol{K}_0 \end{bmatrix}^{\mathsf{T}}.$$

Furthermore, let $G(s; \boldsymbol{K})$ denote the transfer matrix of (9) from w to z, and define

$$g_\lambda(s; \boldsymbol{K}) := S\left(sI - (A_\lambda + B\boldsymbol{K})\right)^{-1} R. \tag{11}$$

Then, it follows that

$$\begin{aligned} \|G(s; \boldsymbol{K})\|_\infty &= \max_{\lambda\in\mathrm{spec}(L)} \|g_\lambda(s; \boldsymbol{K})\|_\infty, \\ \|G(s; \boldsymbol{K})\|_2 &= \sqrt{\textstyle\sum_{\lambda\in\mathrm{spec}(L)} \|g_\lambda(s; \boldsymbol{K})\|_2^2}. \end{aligned} \tag{12}$$

Proof. Let N denote the size of L. With e_i^N denoting the ith column of the N-dimensional identity matrix, we define the permutation matrix

$$\Pi := [I \otimes e_1^N, I \otimes e_2^N, \dots, I \otimes e_N^N],$$

which is unitary. Then, it follows that

$$\Pi^{\mathsf{T}} \widehat{\boldsymbol{A}} \Pi = I \otimes \boldsymbol{A} - L \otimes E.$$

Furthermore, denoting the unitary eigenvector matrix of L by V, we have

$$(V \otimes I)^{\mathsf{T}} \Pi^{\mathsf{T}} \widehat{\boldsymbol{A}} \Pi (V \otimes I) = \mathrm{diag}(\boldsymbol{A} - \lambda E)_{\lambda \in \mathrm{spec}(L)}.$$

Note that $\boldsymbol{A} - \lambda E$ can be rewritten as $A_\lambda + B\boldsymbol{K}$. This proves (10).

Furthermore, we see that

$$\begin{aligned} (V \otimes I)^{\mathsf{T}} \Pi^{\mathsf{T}} (R \otimes I) \Pi &= (I \otimes R)(V \otimes I)^{\mathsf{T}}, \\ \Pi^{\mathsf{T}} (S \otimes I) \Pi (V \otimes I) &= (V \otimes I)(I \otimes S). \end{aligned}$$

Since $\tilde{V} := V \otimes I$ is unitary, we have

$$\begin{aligned} \|G(s; \boldsymbol{K})\|_p &= \|\tilde{V} \mathrm{diag}(g_\lambda(s; \boldsymbol{K}))_{\lambda \in \mathrm{spec}(L)} \tilde{V}^{\mathsf{T}}\|_p \\ &= \|\mathrm{diag}(g_\lambda(s; \boldsymbol{K}))_{\lambda \in \mathrm{spec}(L)}\|_p, \end{aligned}$$

which is valid for both $p \in \{2, \infty\}$. This leads to (12). □

Lemma 1 shows that the stability analysis of $\widehat{\boldsymbol{A}}$ in (7) can be reduced to that of $A_\lambda + B\boldsymbol{K}$, where λ is a nonnegative real parameter varying in $\mathrm{spec}(L)$. The particular structures of A_λ and B lead to the following necessary and sufficient condition on the stability of the distributed control system.

Theorem 2. With the same notation as that in Lemma 1, $\widehat{\boldsymbol{A}}$ is stable if and only if $A_\lambda + B\boldsymbol{K}$ is stable for $\lambda = 0$, whose stability condition is equivalently given by

$$\boldsymbol{K}_0 \boldsymbol{K}_1 \boldsymbol{K}_2 + \boldsymbol{K}_2^2 - \boldsymbol{K}_3 \boldsymbol{K}_0^2 < 0 \tag{13}$$

and $\boldsymbol{K}_j < 0$ for all $j \in \{0, 1, 2, 3\}$.

Proof. From Lemma 1, we see that $\widehat{\boldsymbol{A}}$ is stable if and only if $A_\lambda + B\boldsymbol{K}$ is stable for all $\lambda \in \mathrm{spec}(L)$. In the following, the Routh-Hurwitz stability criterion is to be considered for $A_\lambda + B\boldsymbol{K}$. First, we see that

$$\det sI - (A_\lambda + B\boldsymbol{K}) = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

where the coefficients are given as

$$\begin{aligned} a_1 &= 2\lambda - \boldsymbol{K}_0, & a_2 &= \lambda^2 - 2\boldsymbol{K}_0 \lambda - \boldsymbol{K}_1, \\ a_3 &= -\boldsymbol{K}_0 \lambda^2 - \boldsymbol{K}_1 \lambda - \boldsymbol{K}_2, & a_4 &= -\boldsymbol{K}_3. \end{aligned}$$

Note that all coefficients a_i are positive for any value of $\lambda \in \mathrm{spec}(L)$, which is a subset of nonnegative real numbers including the origin, if and only if all gains $\boldsymbol{K}_i$ are negative.

For simplicity of notation, let $\boldsymbol{F}_i := -\boldsymbol{K}_i$, which is to be positive. Let H_i denote the ith leading principal minor of

$$\begin{bmatrix} a_1 & a_3 & 0 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & 1 & a_2 & a_4 \end{bmatrix}.$$

We show that $H_i > 0$ for all $i \in \{1, 2, 3, 4\}$ if and only if (13) holds. Clearly, $H_1 = a_1 > 0$. Next, we have

$$H_2 = 2\lambda^3 + 4\boldsymbol{F}_0 \lambda^2 + (2\boldsymbol{F}_0^2 + \boldsymbol{F}_1)\lambda + \boldsymbol{F}_0 \boldsymbol{F}_1 - \boldsymbol{F}_2.$$

From the positivity of $\boldsymbol{F}_i$, $H_2 > 0$ for all λ if and only if

$$\boldsymbol{F}_0 \boldsymbol{F}_1 - \boldsymbol{F}_2 > 0. \tag{14}$$

Furthermore, for $H_3 = \sum_{k=0}^{5} b_{5-k} \lambda^k$, we have

$$\begin{aligned} b_0 &= 2\boldsymbol{F}_0, & b_1 &= 4\boldsymbol{F}_0^3 + 2\boldsymbol{F}_1, \\ b_2 &= 2\boldsymbol{F}_0^3 + 5\boldsymbol{F}_0 \boldsymbol{F}_1 + 2\boldsymbol{F}_2, \end{aligned}$$

which are all positive, and

$$\begin{aligned} b_3 &= \boldsymbol{F}_1^2 + 3\boldsymbol{F}_0^2 \boldsymbol{F}_1 + 3\boldsymbol{F}_0 \boldsymbol{F}_2 - \boldsymbol{F}_3, \\ b_4 &= \boldsymbol{F}_0 \boldsymbol{F}_1^2 + 2\boldsymbol{F}_0^2 \boldsymbol{F}_2 - 4\boldsymbol{F}_0 \boldsymbol{F}_3, \\ b_5 &= \boldsymbol{F}_0 \boldsymbol{F}_1 \boldsymbol{F}_2 - \boldsymbol{F}_2^2 - \boldsymbol{F}_0^2 \boldsymbol{F}_3, \end{aligned}$$

each of whose signs is to be investigated. Note that $b_5 > 0$ is necessary to attain $H_2 > 0$ when $\lambda = 0$. Furthermore, from

$$b_5 = \boldsymbol{F}_2(\boldsymbol{F}_0 \boldsymbol{F}_1 - \boldsymbol{F}_2) - \boldsymbol{F}_0^2 \boldsymbol{F}_3 < \boldsymbol{F}_2(\boldsymbol{F}_0 \boldsymbol{F}_1 - \boldsymbol{F}_2),$$

we see that $b_5 > 0$ is sufficient for (14). Next, we show that both b_3 and b_4 are positive if $b_5 > 0$. Dividing $b_5 > 0$ by $\boldsymbol{F}_0$, we have

$$\boldsymbol{F}_0 \boldsymbol{F}_3 < \boldsymbol{F}_1 \boldsymbol{F}_2 - \frac{\boldsymbol{F}_2^2}{\boldsymbol{F}_0},$$

which leads to

$$b_4 > -4\boldsymbol{F}_1 \boldsymbol{F}_2 + 2\boldsymbol{F}_0^2 \boldsymbol{F}_2 + \left(\boldsymbol{F}_0 \boldsymbol{F}_1^2 + 4\frac{\boldsymbol{F}_2^2}{\boldsymbol{F}_0} \right).$$

From the inequality of arithmetic and geometric means, we see that

$$\boldsymbol{F}_0 \boldsymbol{F}_1^2 + 4\frac{\boldsymbol{F}_2^2}{\boldsymbol{F}_0} \geq 4\boldsymbol{F}_1 \boldsymbol{F}_2.$$

Thus, $b_4 > 2\boldsymbol{F}_0^2 \boldsymbol{F}_2 > 0$. Furthermore, we see that

$$b_3 = \frac{b_4}{\boldsymbol{F}_0} + \boldsymbol{F}_0 \boldsymbol{F}_2 + 3\boldsymbol{F}_3 > 0.$$

Hence, $H_3 > 0$ for all λ if and only if $b_5 > 0$, which is identical to (13). Finally, $H_4 = a_4 H_3 > 0$.

Note that the conditions of (13) and $\boldsymbol{K}_j < 0$ are derived when we give $\lambda = 0$. Thus, the stability of $\widehat{\boldsymbol{A}}$ is equivalent to that of $A_\lambda + B\boldsymbol{K}$ for $\lambda = 0$. This proves the claim. □

Theorem 2 shows that the problem of finding a stabilizing parameter $\boldsymbol{K}$ can be equivalently reduced to the problem of designing the *static state feedback gain* for $A_\lambda + B\boldsymbol{K}$ with $\lambda = 0$. Since there can be found a parameter set such that (13) holds, the feasibility of the stabilization problem for the network system $\{\Sigma_i\}_{i \in \mathcal{N}}$ in (2) is always guaranteed, despite imposing a particular structure on the distributed controller $\{C_i\}_{i \in \mathcal{N}}$ in (5).

For the design of $\boldsymbol{K}$ that attains a control specification in terms of the H_∞-norm, we show the following result.

Theorem 3. With the same notation as that in Lemma 1, there exists $\boldsymbol{K}$ such that

$$\|G(s; \boldsymbol{K})\|_\infty < \gamma \tag{15}$$

and G is stable if and only if there exist $\boldsymbol{X} \succ 0$ and $\boldsymbol{Y}$ such that

$$\begin{bmatrix} A_\lambda \boldsymbol{X} + \boldsymbol{X} A_\lambda^{\mathsf{T}} + B\boldsymbol{Y} + \boldsymbol{Y}^{\mathsf{T}} B^{\mathsf{T}} + RR^{\mathsf{T}} & * \\ S\boldsymbol{X} & -\gamma^2 I \end{bmatrix} \prec 0 \tag{16}$$

for both $\lambda \in \{0, \lambda_{\max}\}$, where $\lambda_{\max}$ denotes the maximal eigenvalue of L. Furthermore, if (16) is feasible, then

$$\boldsymbol{K} = \boldsymbol{Y} \boldsymbol{X}^{-1} \tag{17}$$

satisfies (15).

Proof. As shown in [Zhou et al. (1996)], on the premise of the change of variables in (17), it is known that the feasibility of (16) is equivalent to

$$\|g_\lambda(s;\boldsymbol{K})\|_\infty < \gamma, \tag{18}$$

where g_λ is defined as in (11). Since (12) holds, to prove the claim, it suffices to show that (18) holds for all $\lambda \in \mathrm{spec}(L)$ if and only if it holds for $\lambda \in \{0, \lambda_{\max}\}$. This is proven by the facts that A_λ is affine with respect to λ, and there exists some $p \in [0,1]$ such that

$$\lambda = 0 \cdot p + (1-p)\lambda_{\max}$$

for each of $\lambda \in \mathrm{spec}(L)$. □

Theorem 3 shows that the controller parameter $\boldsymbol{K}$ that attains the H_∞-control specification in (15) can be found by calculating $\boldsymbol{X}$ and $\boldsymbol{Y}$ that simultaneously solve the system of linear matrix inequalities in (16) for $\lambda \in \{0, \lambda_{\max}\}$. An analysis in terms of the H_2-norm can also be done in a similar manner.

3.3 Discussion

To make our contribution clearer, we provide a brief comparison with a distributed controller design method for network systems, composed of identical subsystems. For convenience, only in this subsection, we use notation isolated from the other parts.

Let us consider the class of network systems described as

$$\begin{cases} \dot{x} = (I \otimes A + L \otimes E)x + (I \otimes B)u \\ y = (I \otimes C)x \end{cases}$$

where x is the state, u is the external input signal, and y is the measurement output signal. To this network system, we implement a distributed controller given as

$$\begin{cases} \dot{\xi} = (I \otimes \boldsymbol{K} + L \otimes \boldsymbol{G})\xi + (I \otimes \boldsymbol{H})y \\ u = (I \otimes \boldsymbol{F})\xi, \end{cases}$$

which has the same interconnection structure as the network system. The matrices in the bold face are the controller parameters.

Let V denote the eigenvector matrix of L. By the coordinate transformation by $V \otimes I$, which simultaneously makes the network structures of the system and controller disjoint, we see that the entire closed-loop system is stable if and only if

$$\begin{bmatrix} A + \lambda E & B\boldsymbol{F} \\ \boldsymbol{H}C & \boldsymbol{K} + \lambda \boldsymbol{G} \end{bmatrix}$$

is stable for all $\lambda \in \mathrm{spec}(L)$. In fact, the stabilizing controller design problem is decoupled into a set of small problems. However, each of them is a design problem of dynamical feedback controllers, whose dynamics involves the parameter $\lambda \in \mathrm{spec}(L)$ as well. The same difficulty applies even when the local state feedback is allowed, i.e., $C = I$. This clearly contrasts with our result, in which the distributed controller design problem is reduced to a static state feedback design problem.

4. NUMERICAL SIMULATION

In this section, we show the efficiency of our distributed control system design method by numerical simulations. For the air conditioning system in (1), we use $\tau_i = 1$ [min], $c_i = 164.9$ [kJ/K], and $k_{i,j} = 34.35$ [kJ/K·min], which are obtained by an experiment of system identification. For this system, we consider the two situations: (i) unknown impulsive disturbances and (ii) unknown constant disturbances are injected to Areas 1 to 3 in Fig. 1. The first one is supposed to simulate a situation where three windows attached to Areas 1 to 3 are made open for a short time, while the second is to simulate a situation where constant heat loads, e.g., computing machineries, are placed at those areas.

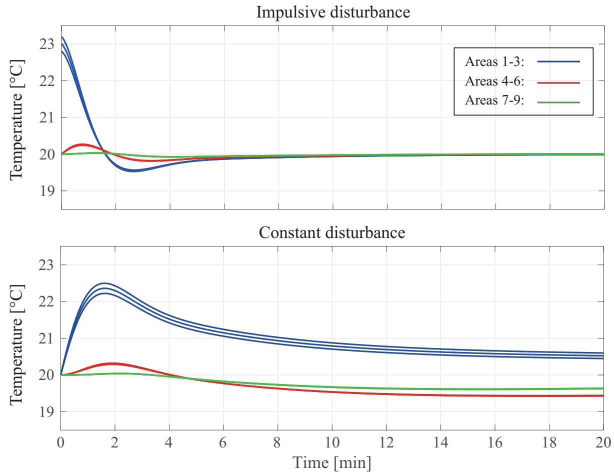

Fig. 3. Temperature regulation for impulsive and constant disturbances.

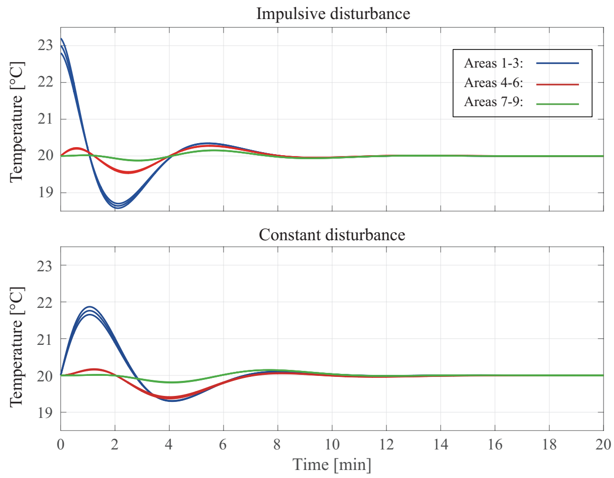

Fig. 4. Temperature regulation for impulsive and constant disturbances.

For the design of distributed controllers, we apply a type of LQR design techniques, which can be formulated as an LMI-based optimization problem in terms of the H_2-norm similar to (16). In particular, to find a feedback gain $\boldsymbol{K}$, we solve an optimization problem of minimizing γ such that

$$\int_0^\infty (e_\lambda^\mathsf{T} Q e_\lambda + v^2)dt < \gamma, \quad \begin{cases} \dot{e}_\lambda = A_\lambda e_\lambda + Bv \\ v = \boldsymbol{K} e_\lambda, \end{cases}$$

for both $\lambda \in \{0, \lambda_{\max}\}$, where A_λ, B, and $\boldsymbol{K}$ are given as in (10). Giving $Q = 0.02 \times \mathrm{diag}(1,1,100,1)$, we have

$$\boldsymbol{K} = [-0.020, -0.451, -2.365, -2.178].$$

The resultant system responses for (i) impulsive and (ii) constant disturbances are shown in Fig. 3, where the temperature variables x_i corresponding to Areas 1 to 3, 4 to 6, and 7 to 9 are denoted by the blue, red, and green lines, respectively, and their reference value is given as $r_i = 20$ [°C]. From this figure, we see that all temperature variables converge to the reference value within around 10 [min] in the case of the impulsive disturbances, while their convergence rate for the constant disturbances is not sufficient. This stems from the fact that the weight of Q corresponding to the temperature variable x_i is relatively large, while that corresponding to the integrator η_i is small.

To improve the convergence rate for the constant disturbances, we give the weight as $Q = 0.02 \times \mathrm{diag}(100, 1, 1, 1)$, which leads to

$$\boldsymbol{K} = [-1.040, -2.540, -3.576, -2.615].$$

The resultant system responses are shown in Fig. 4. From this figure, we see that the convergence rate for the constant disturbances is higher but the transient response for the impulsive disturbances is more oscillatory than those in Fig. 3. This kind of degree of freedom in controller design stems from our success that the design problem of distributed controllers is reduced to that of static state feedback design, which can be systematically solved by applying existing controller design methods.

5. CONCLUSION

In this paper, we have proposed a design method of distributed temperature regulators for an air conditioning system, which can steer room temperatures to a reference value under the existence of unknown constant heat loads. For the distributed controller design, we have derived a necessary and sufficient condition of stabilizing controller parameters, while confining the class of controllers to that consisting of subcontrollers having a network structure identical to that of the controlled system. Furthermore, we have shown that the problem of distributed controller design can be reduced to a problem of robust state feedback gain design, which can be systematically solved by applying the existing controller design methods. The efficiency of our distributed regulator design method has been shown through numerical simulations, in which we have considered two distributed controllers specializing to the stabilization of impulsive and constant disturbances. The experimental verification of the efficiency of our distributed controller design method would be one of future works to pursue.

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