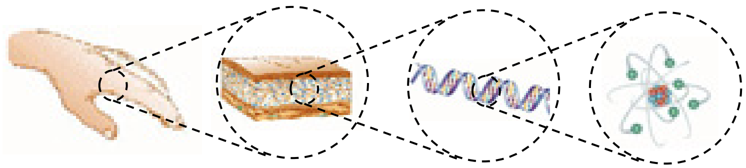
Seminar at KTH Royal Institute of Technology

Multiresolved Control of Discrete-Time Linear Systems

Takayuki Ishizaki (Tokyo Institute of Technology)

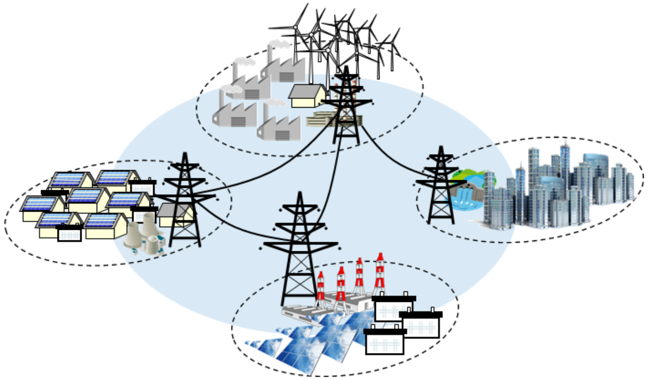
Motivation

Biological systems



Organs, cells, proteins, …
are interacting hierarchically

Power system control



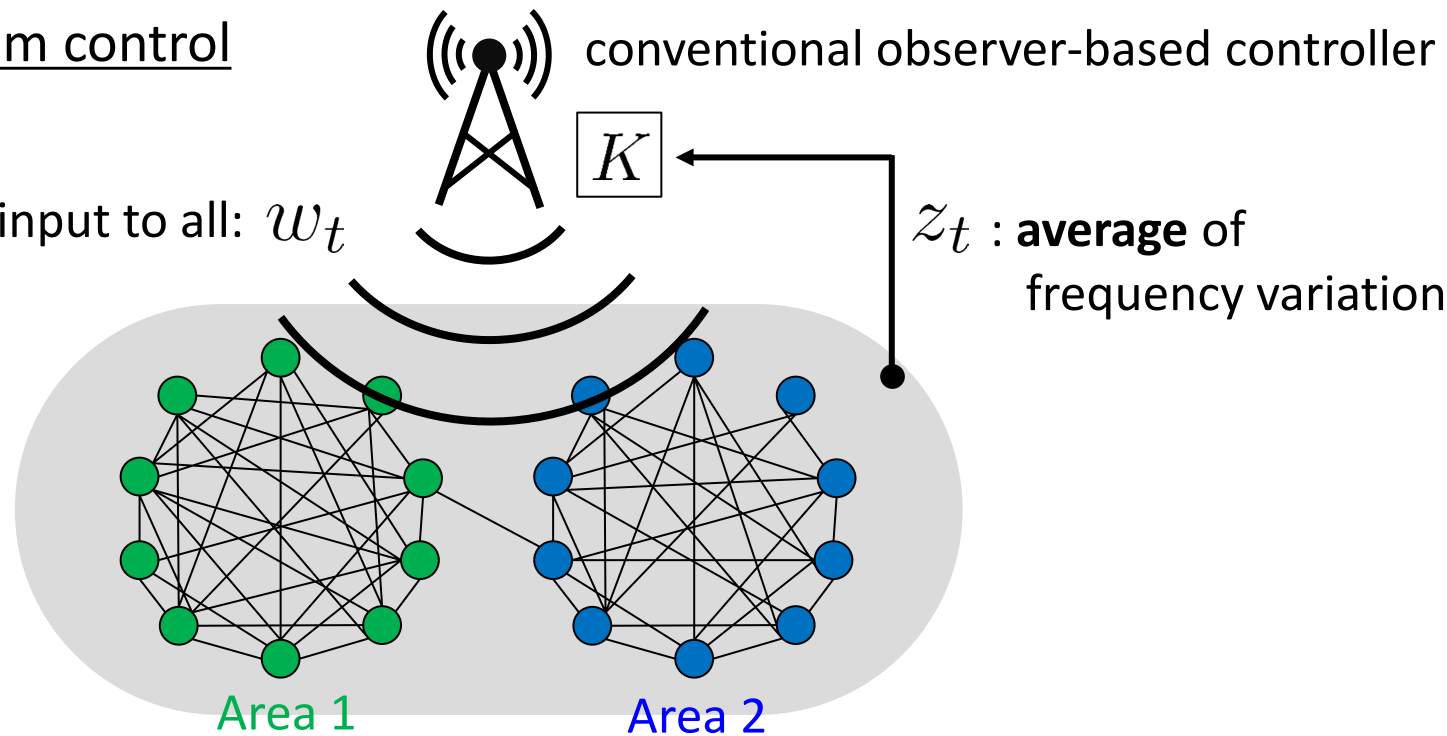
Few minutes: governor free control
Few dozen minutes: LFC, EDC

Efficient control of systems on different spatiotemporal scales?

Multiresolved control: Utilize global/local properties in time and space

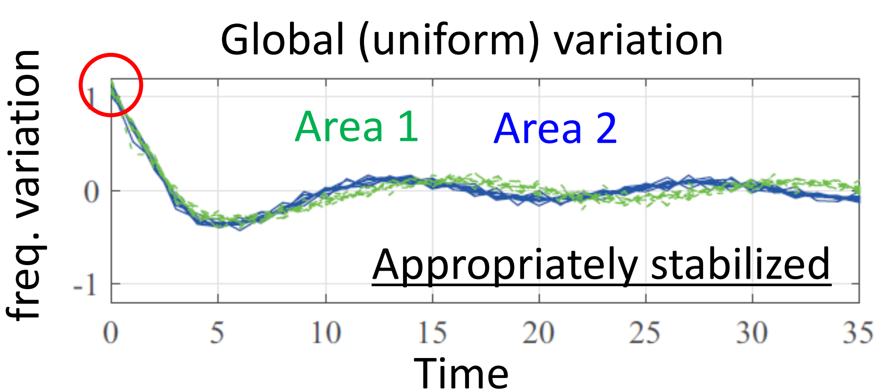
A Multiresolved Control System

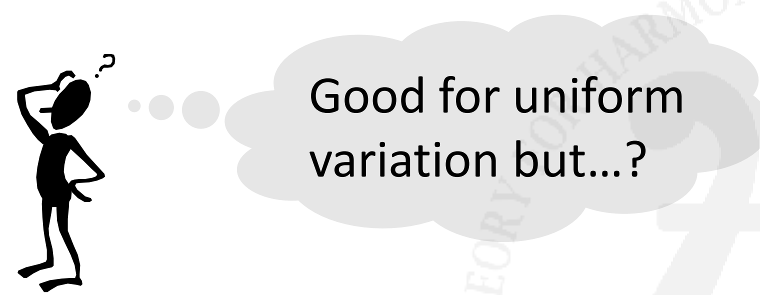
<u>ex) Power system control</u>



: generators

easy to synchronize
in each area





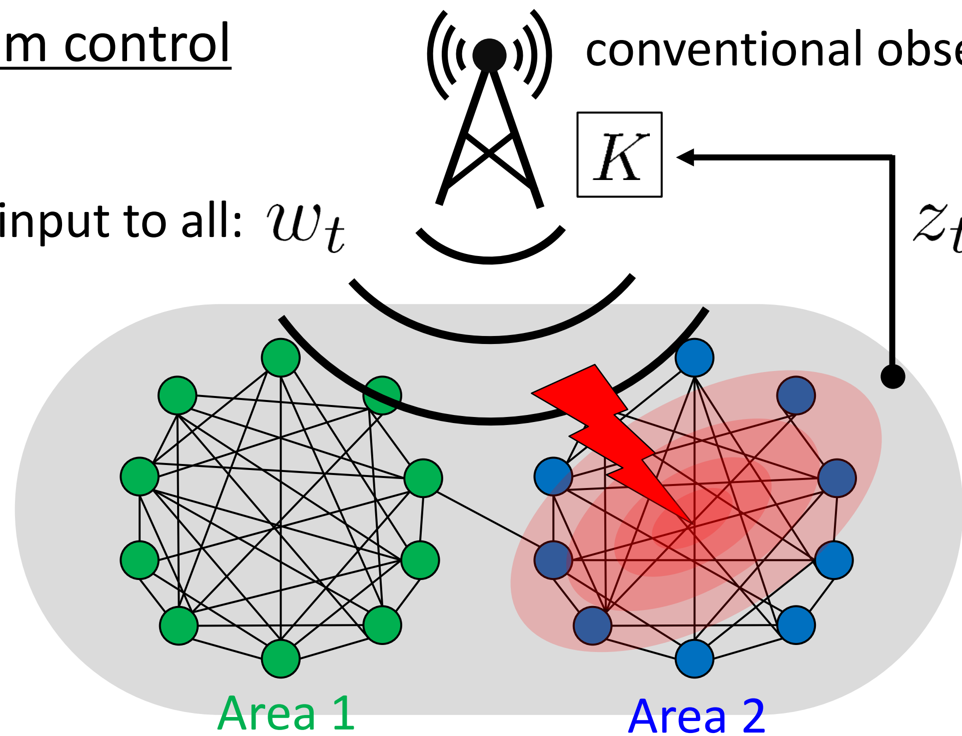
A Multiresolved Control System

ex) Power system control

conventional observer-based controller

identical input to all: w_t

z_t : **average** of frequency variation

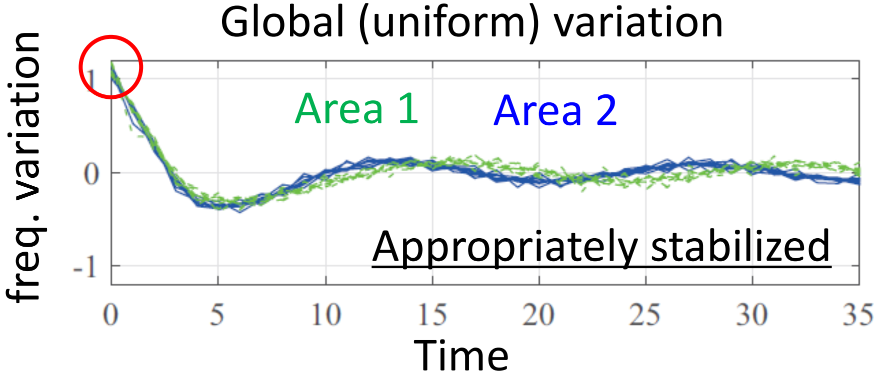


: generators

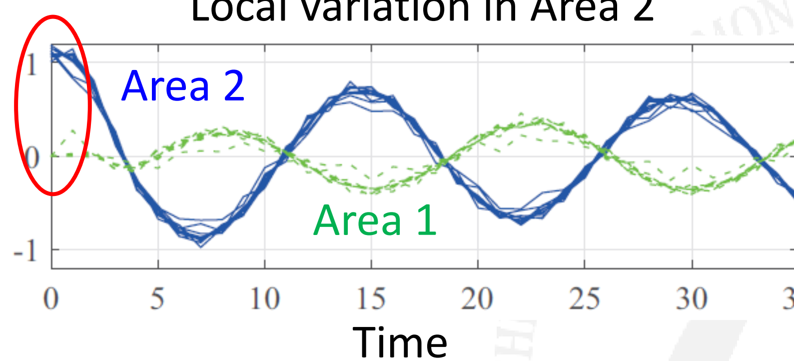
easy to synchronize in each area

not suitable for local (nonuniform) variation

Global (uniform) variation

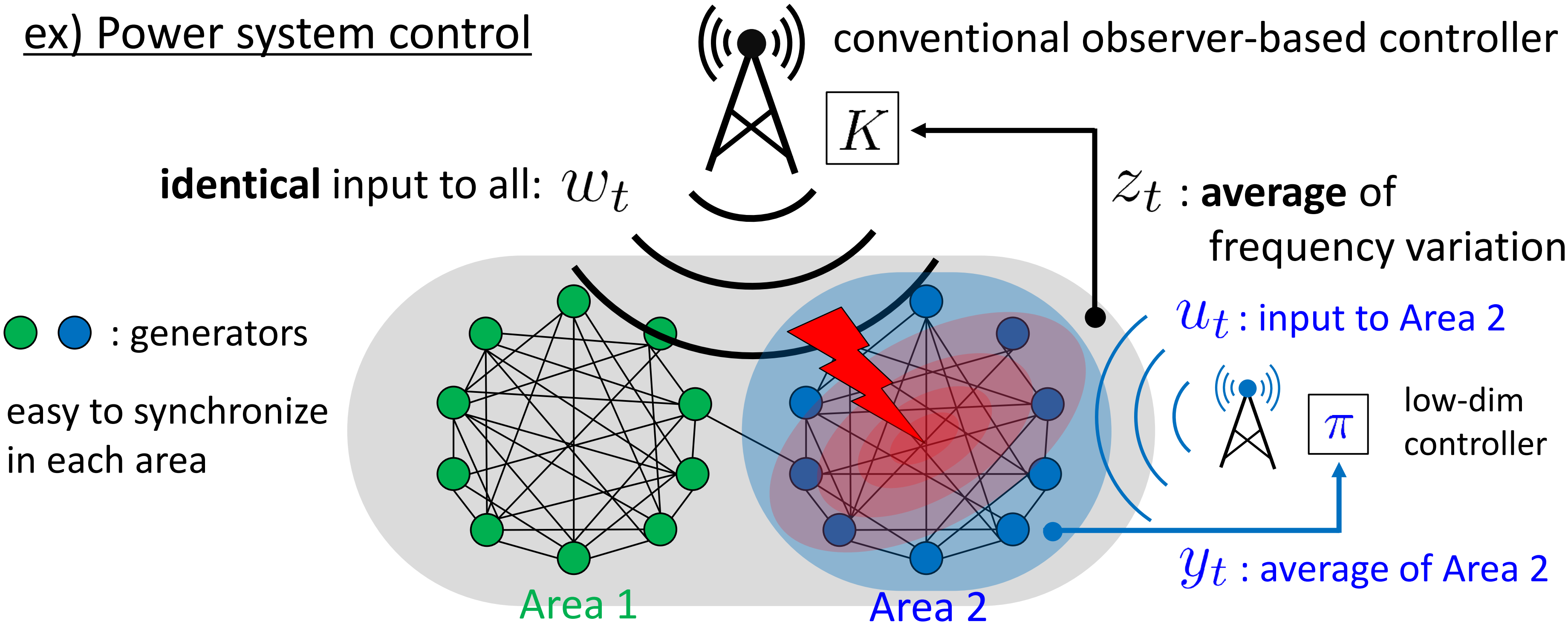


Local variation in Area 2

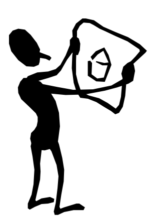


A Multiresolved Control System

ex) Power system control



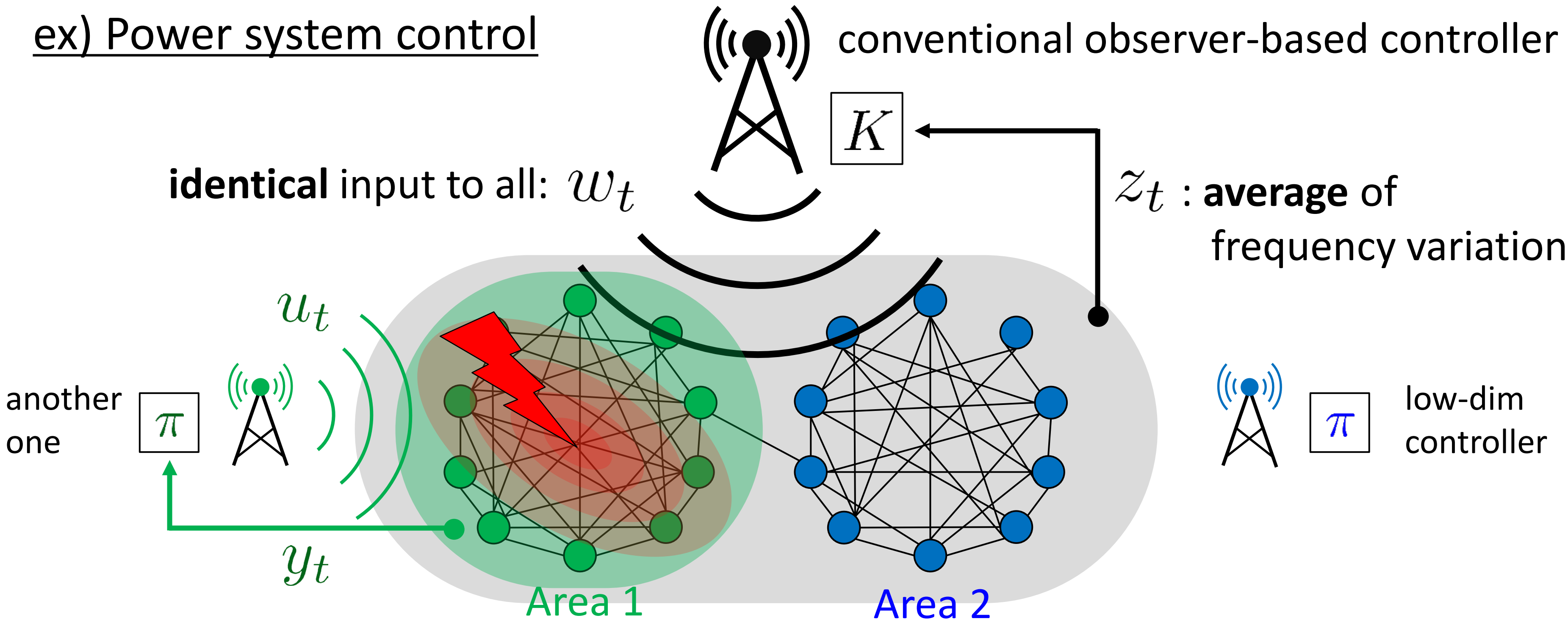
Design a low-dim local controller π cooperating with a given K



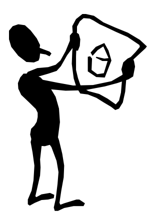
- Plug-in when and where necessary
- Local action for a specific time interval

A Multiresolved Control System

ex) Power system control



Design a low-dim local controller π cooperating with a given K



- Plug-in when and where necessary
- Local action for a specific time interval

Outline

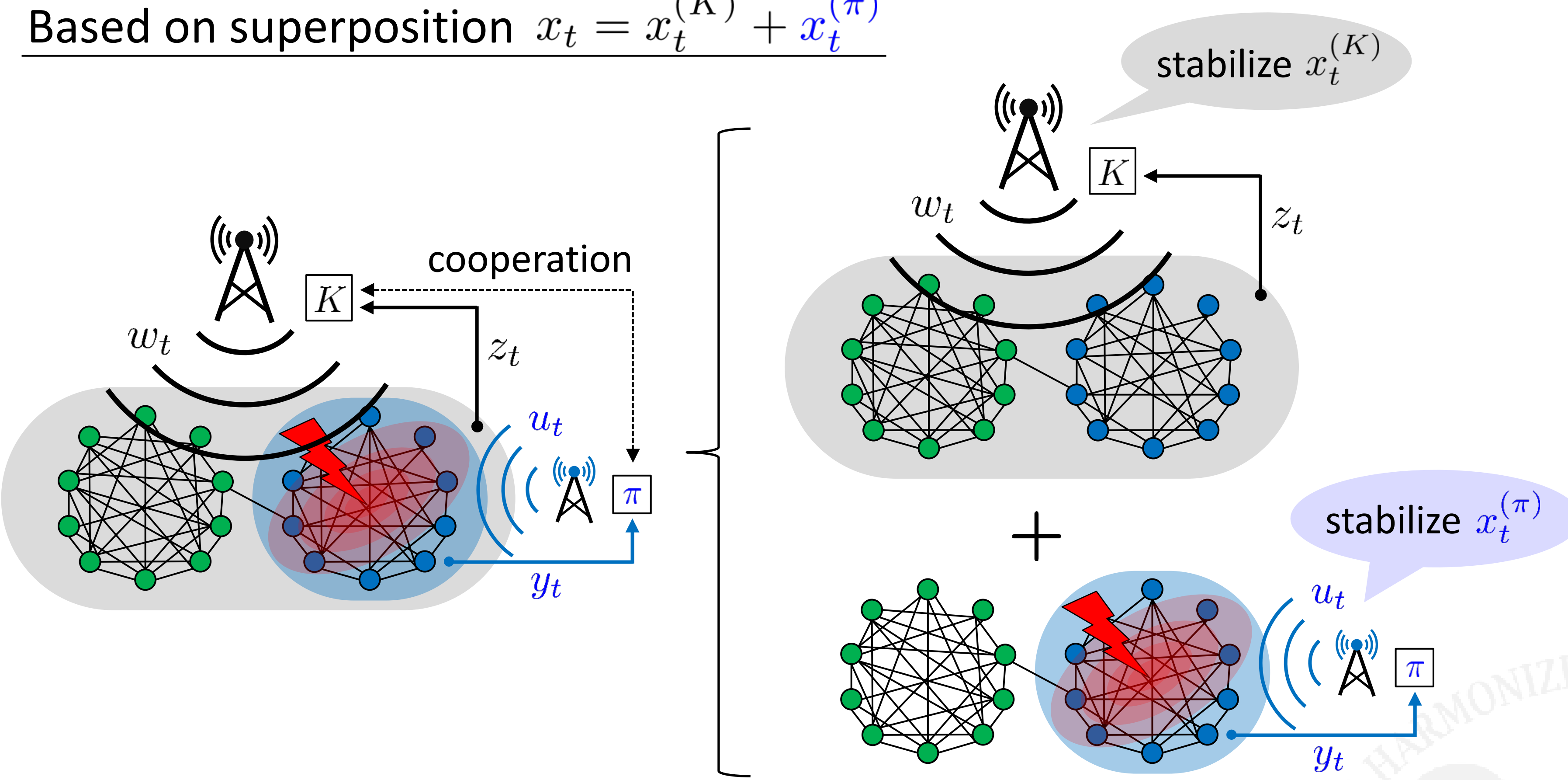
- Motivation

- Key idea: Superposition by state-space expansion
- Main result: Resultant multiresolved control system

- Technical details
- Numerical example

How to systematically design π ?

Based on superposition $x_t = x_t^{(K)} + x_t^{(\pi)}$

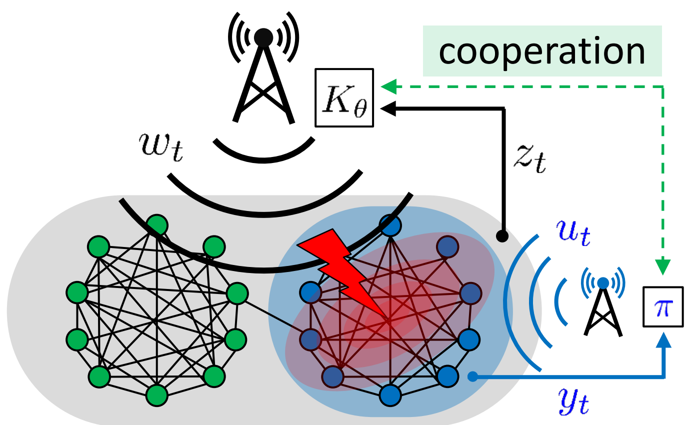


Superposition by state-space expansion

Main Result

【Theorem】

Each of $x_t^{(K)}, x_t^{(\pi)}$ is stabilized by



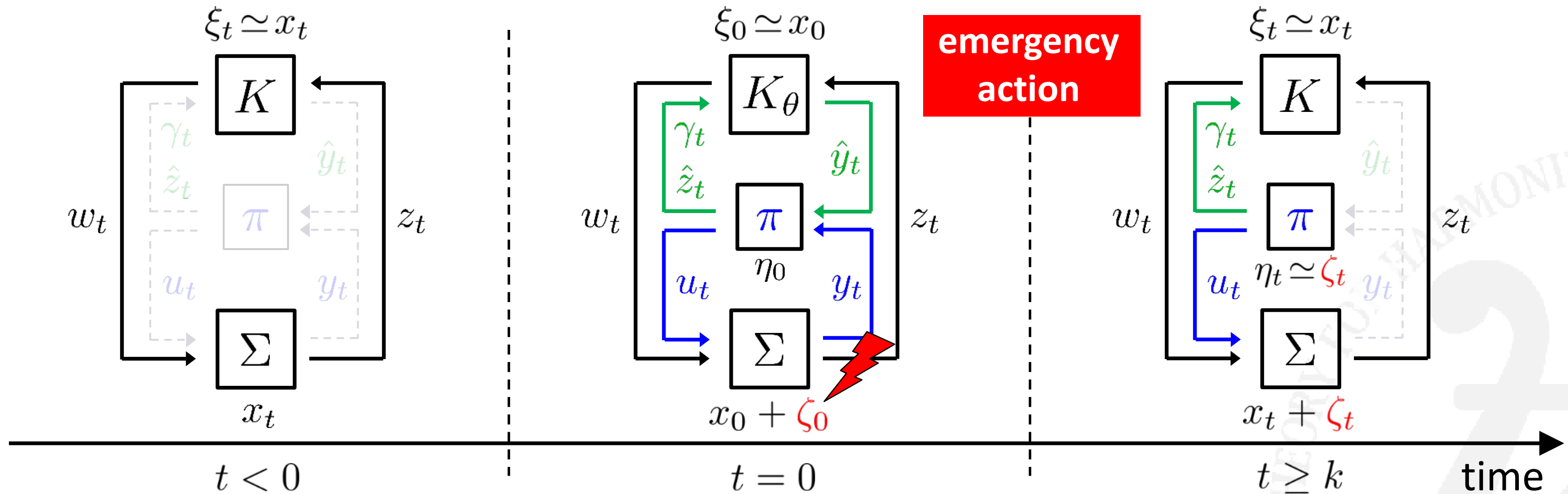
$$
K_\theta : \begin{cases} \xi_{t+1} &= (A+RF)\xi_t + H(z_t - \hat{z}_t - S\xi_t) + \phi_k \gamma_t \\ \theta_{t+1} &= \hat{A}_\theta \theta_t + \hat{B}_\theta (z_t - \hat{z}_t - S\xi_t) \\ \hat{y}_t &= C\xi_t - \hat{C}_\theta \theta_t \\ w_t &= F\xi_t \end{cases}
$$

n-dim controller + k-dim memory

$$
\pi : \begin{cases} \eta_{t+1} &= (\hat{A} + \hat{B}\hat{F})\eta_t + \hat{H}(y_t - \hat{y}_t - \hat{C}\eta_t) \\ \gamma_t &= e_k^\mathsf{T} \eta_t \\ \hat{z}_t &= \hat{S}\eta_t \\ u_t &= \hat{F}\eta_t \end{cases}
$$

k-dim local controller

✓ $\hat{H} = 0$ after time k



Outline

- Motivation

- Key idea: Superposition by state-space expansion
- Main result: Resultant multiresolved control system

- Technical details
- Numerical example

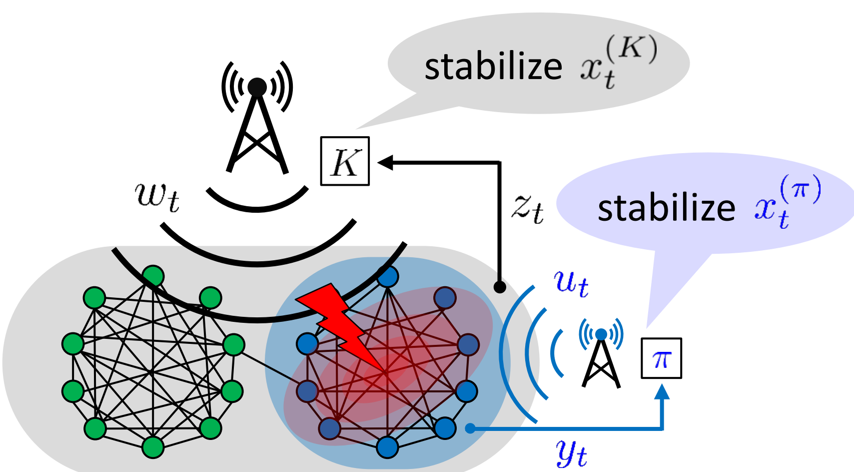
Superposition by State-Space Expansion

block triangular

State-space expansion

$$\Sigma : \begin{cases} x_{t+1} &= Ax_t + B{\color{blue}u_t} + Rw_t \\ z_t &= Sx_t \\ {\color{blue}y_t} &= Cx_t \end{cases} \quad \underline{n\text{-dim}} \qquad \Leftrightarrow \qquad \tilde{\Sigma} : \begin{cases} \begin{bmatrix} x_{t+1}^{(K)} \\ x_{t+1}^{(\pi)} \end{bmatrix} &= \begin{bmatrix} A & A - {\color{red}\hat{A}} \\ 0 & {\color{red}\hat{A}} \end{bmatrix} \begin{bmatrix} x_t^{(K)} \\ x_t^{(\pi)} \end{bmatrix} + \begin{bmatrix} Rw_t \\ B{\color{blue}u_t} \end{bmatrix} \\ z_t &= Sx_t^{(K)} + Sx_t^{(\pi)} \\ {\color{blue}y_t} &= Cx_t^{(K)} + Cx_t^{(\pi)} \end{cases} \quad \underline{2n\text{-dim}}$$

【Lemma】 $x_t = x_t^{(K)} + x_t^{(\pi)}$ for any $w_t, {\color{blue}u_t}$ if $x_0 = x_0^{(K)} + x_0^{(\pi)}$



Decentralized state FB: $\begin{cases} w_t &= F^{(K)} x_t^{(K)} \\ {\color{blue}u_t} &= {\color{blue}F^{(\pi)} x_t^{(\pi)}} \end{cases}$

$x_t^{(K)}, {\color{blue}x_t^{(\pi)}}$ are not directly measurable…

Decentralized observation of $x_t^{(K)}, {\color{blue}x_t^{(\pi)}}$

Decentralized Observation of Superposition States?

Redundant realization ($2n$-dim) $\quad x_0 = \hat{x}_0 + \zeta_0$

unknown local variation

$$\tilde{\Sigma}: \begin{cases} \begin{bmatrix} x_{t+1}^{(K)} \\ x_{t+1}^{(\pi)} \end{bmatrix} = \begin{bmatrix} A & A-\hat{A} \\ 0 & \hat{A} \end{bmatrix} \begin{bmatrix} x_t^{(K)} \\ x_t^{(\pi)} \end{bmatrix} + \begin{bmatrix} Rw_t \\ Bu_t \end{bmatrix}, \quad \begin{bmatrix} x_0^{(K)} \\ x_0^{(\pi)} \end{bmatrix} = \begin{bmatrix} \hat{x}_0 \\ \zeta_0 \end{bmatrix} \\ z_t = Sx_t^{(K)} + Sx_t^{(\pi)} \\ y_t = Cx_t^{(K)} + Cx_t^{(\pi)} \end{cases}$$

Luenberger–type observer

$$\tilde{\Sigma}_{\mathrm{obs}}: \begin{cases} \begin{bmatrix} \hat{x}_{t+1}^{(K)} \\ \hat{x}_{t+1}^{(\pi)} \end{bmatrix} = \begin{bmatrix} A & A-\hat{A} \\ 0 & \hat{A} \end{bmatrix} \begin{bmatrix} \hat{x}_t^{(K)} \\ \hat{x}_t^{(\pi)} \end{bmatrix} + \begin{bmatrix} Rw_t \\ Bu_t \end{bmatrix} + \begin{bmatrix} H^{(K)}(z_t - \hat{z}_t) \\ H^{(\pi)}(y_t - \hat{y}_t) \end{bmatrix}, \quad \begin{bmatrix} \hat{x}_0^{(K)} \\ \hat{x}_0^{(\pi)} \end{bmatrix} = \begin{bmatrix} \hat{x}_0 \\ 0 \end{bmatrix} \\ \hat{z}_t = S\hat{x}_t^{(K)} + S\hat{x}_t^{(\pi)} \\ \hat{y}_t = C\hat{x}_t^{(K)} + C\hat{x}_t^{(\pi)} \end{cases}$$

decentralized output feedback

Error dynamics

block ~~triangular~~

$$\begin{bmatrix} e_{t+1}^{(K)} \\ e_{t+1}^{(\pi)} \end{bmatrix} = \begin{bmatrix} A - H^{(K)}S & A - \hat{A} - H^{(K)}S \\ -H^{(\pi)}C & \hat{A} - H^{(\pi)}C \end{bmatrix} \begin{bmatrix} e_t^{(K)} \\ e_t^{(\pi)} \end{bmatrix}$$

Can we make $H^{(\pi)}Ce_t^{(K)} = 0$?

Block-Triangulation of Error Dynamics

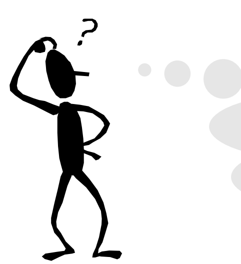
【Lemma】

If
$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} = \begin{bmatrix} C \\ C\hat{A} \\ \vdots \\ C\hat{A}^{k-1} \end{bmatrix}, \quad \begin{bmatrix} CH^{(K)} \\ CAH^{(K)} \\ \vdots \\ CA^{k-1}H^{(K)} \end{bmatrix} = \begin{bmatrix} \hat{C}_\theta \hat{B}_\theta \\ \hat{C}_\theta \hat{A}_\theta \hat{B}_\theta \\ \vdots \\ \hat{C}_\theta \hat{A}_\theta^{k-1} \hat{B}_\theta \end{bmatrix}$$

k-dim Krylov projection model

and $H^{(\pi)} = 0$ after time k, then

$$\begin{bmatrix} \theta_{t+1} \\ e_{t+1}^{(K)} \\ e_{t+1}^{(\pi)} \end{bmatrix} = \begin{bmatrix} \hat{A}_\theta & \hat{B}_\theta S & \hat{B}_\theta S \\ 0 & A - H^{(K)}S & A - \hat{A} - H^{(K)}S \\ \cancel{H^{(\pi)}}\hat{C}_\theta & -\cancel{H^{(\pi)}}C & \hat{A} - H^{(\pi)}C \end{bmatrix} \begin{bmatrix} \theta_t \\ e_t^{(K)} \\ e_t^{(\pi)} \end{bmatrix} \begin{matrix} k\text{-dim} \\ n\text{-dim} \\ n\text{-dim...?} \end{matrix}$$



Some $\hat{A}$ with "lower complexity"
such that $\boxed{\quad}$?

Wedderburn Rank Reduction

【Wedderburn Rank Reduction】 [Chu et al. 1995]

For given $R = [r_1, \ldots, r_n], \quad O = [o_1, \ldots, o_n],$ there exist

$$\begin{cases} U = [u_1, \ldots, u_n] \\ V = [v_1, \ldots, v_n] \end{cases}$$ such that $$\boxed{\begin{cases} \operatorname{im} U = \operatorname{im} R \\ \operatorname{im} V = \operatorname{im} O \end{cases} \quad V^{\mathsf{T}} A U = \operatorname{diag}(\omega_i)}$$ and

rank-one decomposition $$A = \sum_{i=1}^{n} \omega_i^{-1} \phi_i \psi_i^{\mathsf{T}}$$ where $$\begin{cases} \phi_i := A u_i \\ \psi_i := A^{\mathsf{T}} v_i. \end{cases}$$

✓ ▭ : generalization of Gram-Schmidt basis orthogonalization

if $\quad R = O, \quad A = I_n,$ $$\boxed{\operatorname{im} U = \operatorname{im} R, \quad u_i^{\mathsf{T}} u_j = \begin{cases} \omega_i, & i = j \\ 0, & i \neq j \end{cases}}$$

Wedderburn Rank Reduction

【Wedderburn Rank Reduction】 [Chu et al. 1995]

For given $R = [r_1, \ldots, r_n]$, $O = [o_1, \ldots, o_n]$, there exist

$$\begin{cases} U = [u_1, \ldots, u_n] \\ V = [v_1, \ldots, v_n] \end{cases}$$ such that $$\begin{cases} \operatorname{im} U = \operatorname{im} R \\ \operatorname{im} V = \operatorname{im} O \end{cases} \quad V^{\mathsf{T}} A U = \operatorname{diag}(\omega_i)$$ and

$$A = \underbrace{\sum_{i=1}^{k-1} \omega_i^{-1} \phi_i \psi_i^{\mathsf{T}}}_{\hat{A}} + \sum_{i=k}^{n} \omega_i^{-1} \phi_i \psi_i^{\mathsf{T}} \quad \text{where} \quad \begin{cases} \phi_i := A u_i \\ \psi_i := A^{\mathsf{T}} v_i. \end{cases}$$

【Lemma】 If R and O are controllability and observability matrices,

$$[B, AB, \ldots, A^{k-1}B] = [B, \hat{A}B, \ldots, \hat{A}^{k-1}B] \qquad \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} = \begin{bmatrix} C \\ C\hat{A} \\ \vdots \\ C\hat{A}^{k-1} \end{bmatrix}$$

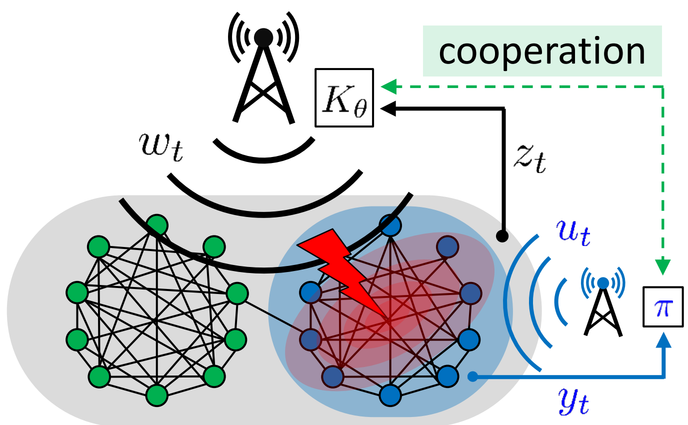
Markov parameter matching

✓ the low-rank $(\hat{A}, B, C)$ is equivalent to a k-dim Krylov projection model

Main Result (reshown)

【Theorem】

Each of $x_t^{(K)}, x_t^{(\pi)}$ is stabilized by



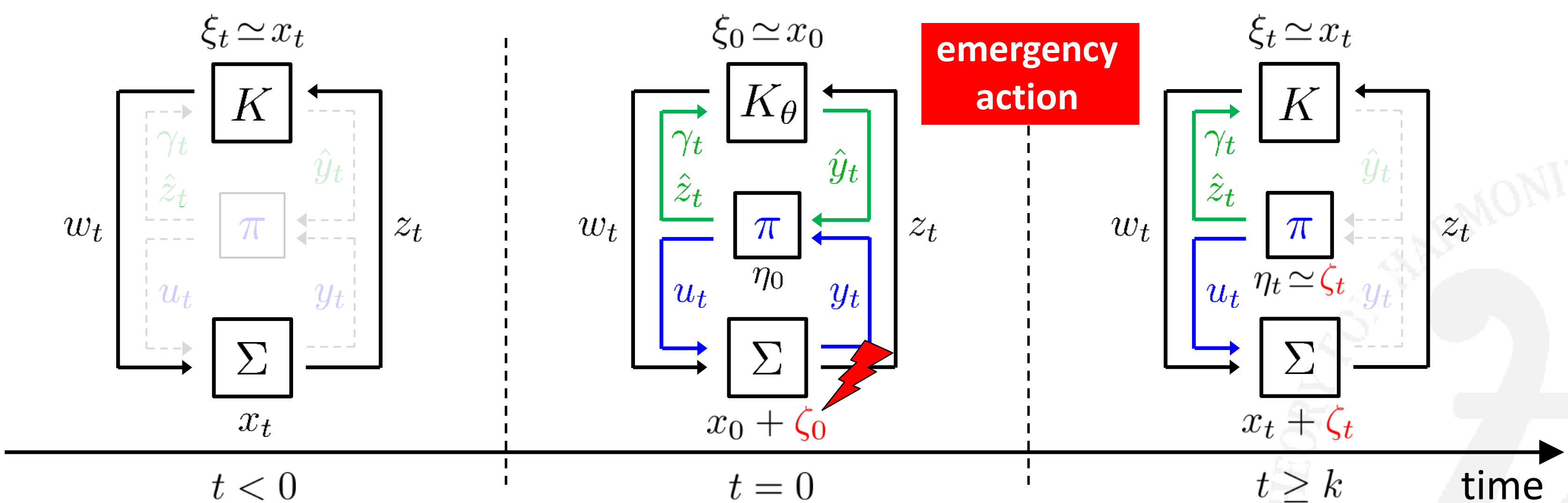
$$K_\theta : \begin{cases} \xi_{t+1} = (A + RF)\xi_t + H(z_t - \hat{z}_t - S\xi_t) + \phi_k \gamma_t \\ \theta_{t+1} = \hat{A}_\theta \theta_t + \hat{B}_\theta (z_t - \hat{z}_t - S\xi_t) \\ \hat{y}_t = C\xi_t - \hat{C}_\theta \theta_t \\ w_t = F\xi_t \end{cases}$$

n-dim controller + k-dim memory

$$\pi : \begin{cases} \eta_{t+1} = (\hat{A} + \hat{B}\hat{F})\eta_t + \hat{H}(y_t - \hat{y}_t - \hat{C}\eta_t) \\ \gamma_t = e_k^\top \eta_t \\ \hat{z}_t = \hat{S}\eta_t \\ u_t = \hat{F}\eta_t \end{cases}$$

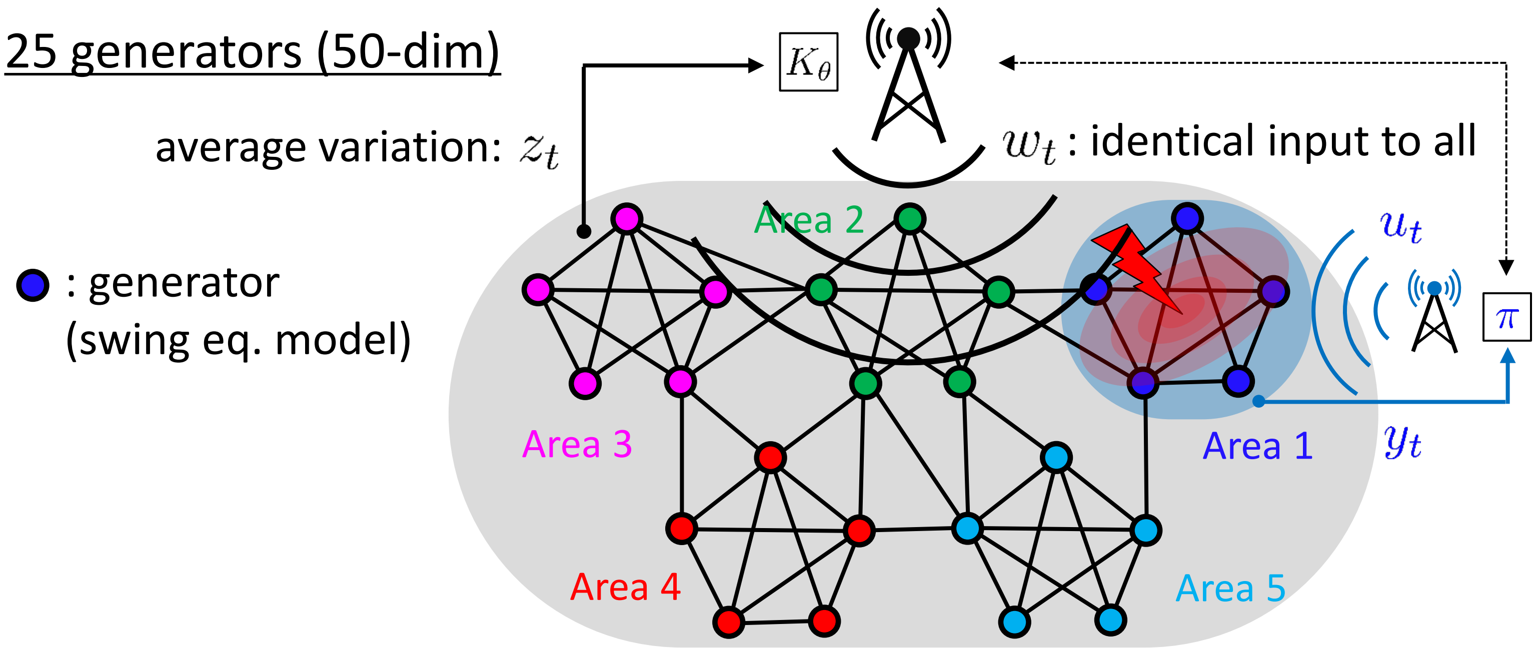
k-dim local controller

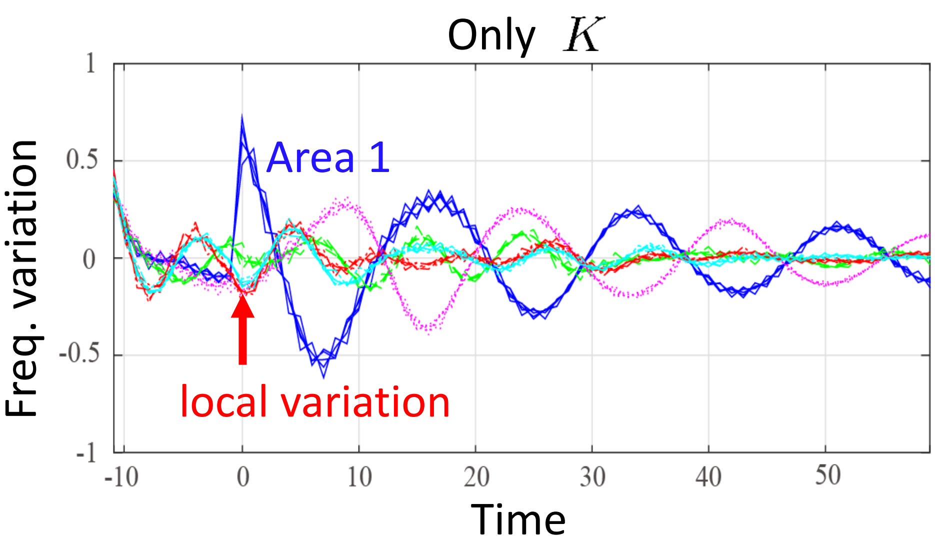
✓ $\hat{H} = 0$ after time k

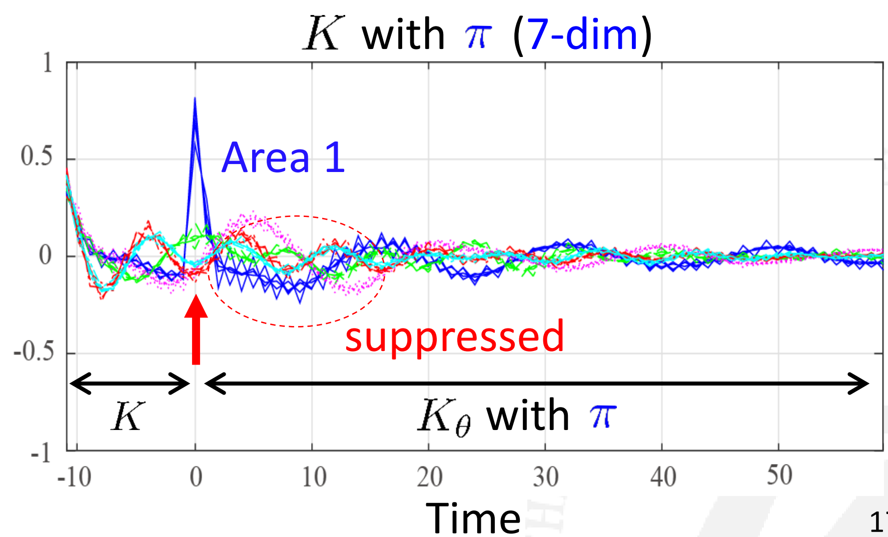


Numerical Example

25 generators (50-dim)







Concluding Remarks

- Multiresolved control
  - state-space expansion (global model + local model)
  - cooperation of global and local controllers
    - global control at all times
    - local control when and where necessary
  - one possible application of model reduction techniques

- Future works
  - multi-layer case (local-middle-global)
  - multi-rate sampling
  - make an appropriate criterion to plug in local controllers

Thank you for your attention!