非負システム調査研究会

ネットワークシステムの階層分散制御と非負システムへの応用の検討

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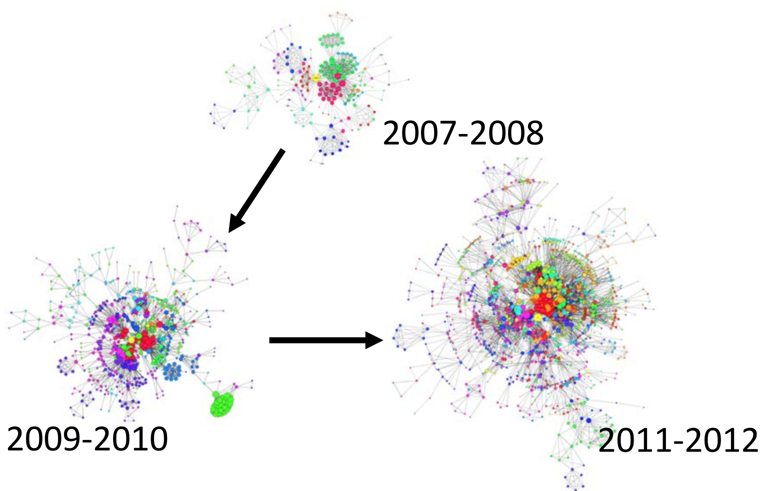
Background

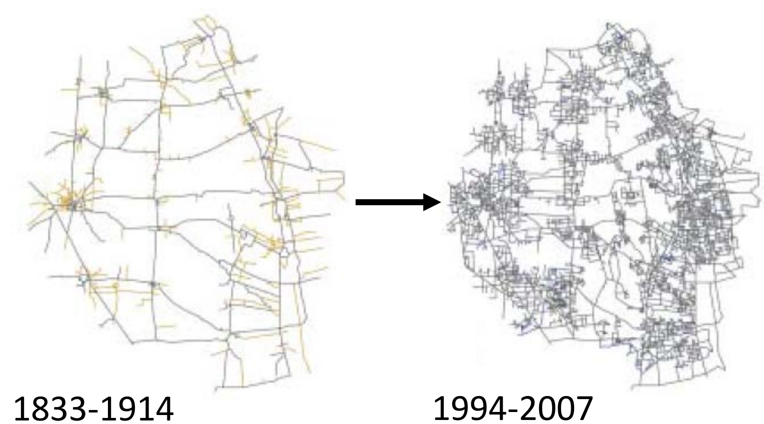
- Distributed control of networked systems
  - Structural constraints [Siljak et al.] Inclusion principle [Ikeda et al.]
  - Specific class: Quadratic invariance [Rotkowitz et al.] Positive systems [Ebihara et al.]
- Real world networks are evolving

Apple's inventor network



http://www.kenedict.com

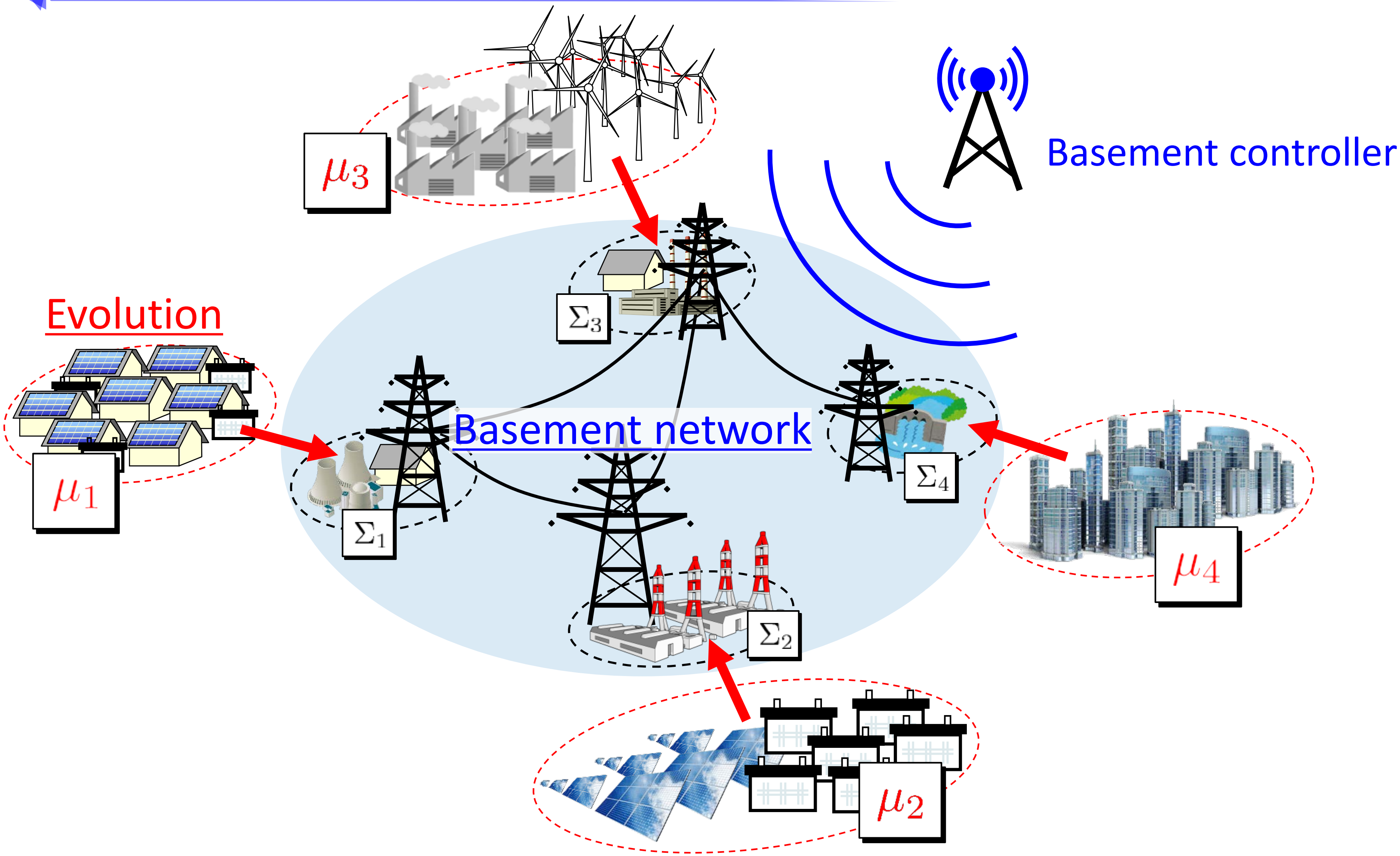
Road network in Milan



[E. Strano et al. 2012]

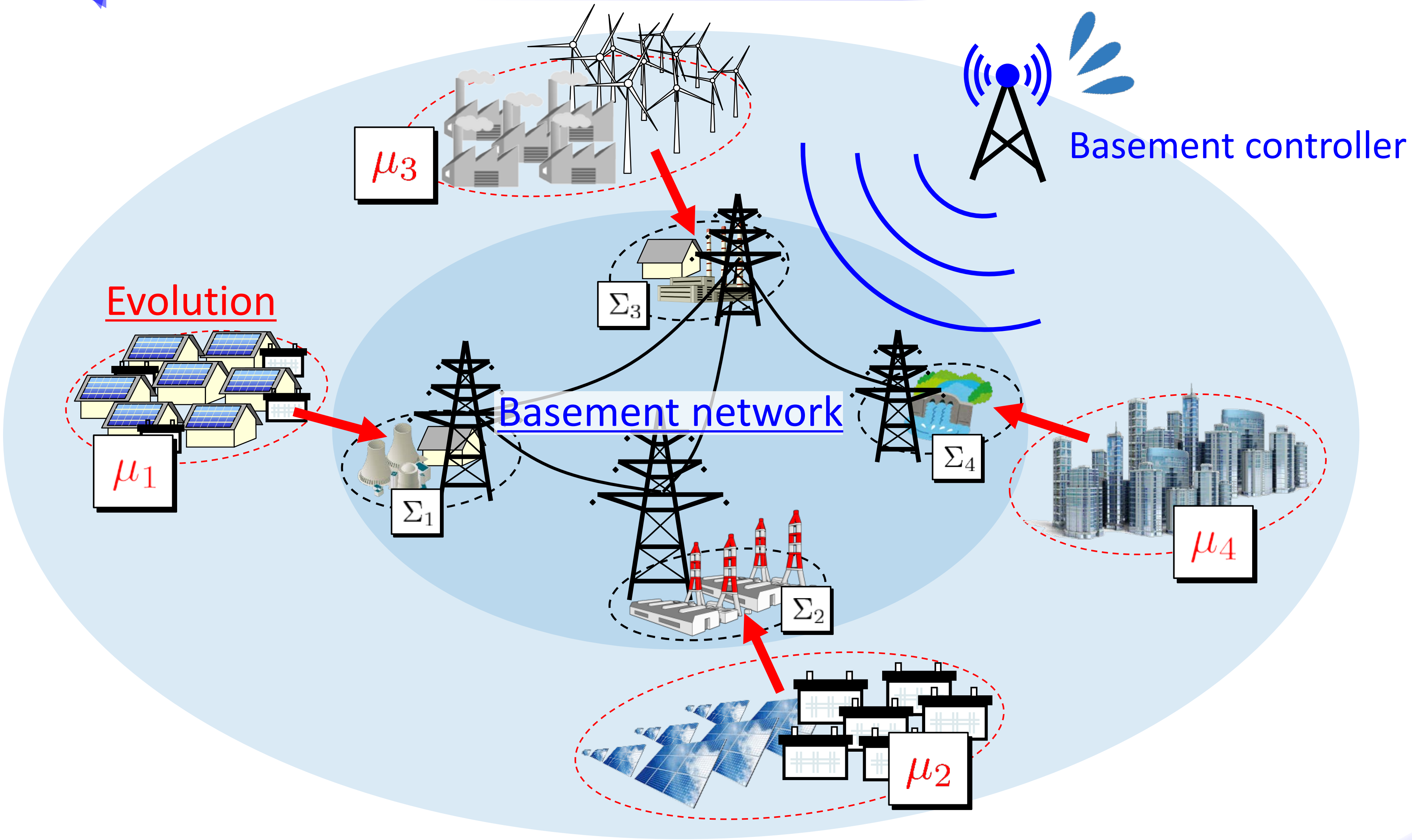
How to deal with **evolving networks** in a tractable manner?

Control of Evolving Networks



$\{\Sigma_i\}^{\rm net}_{\leftrightarrow}$: Basement network (fixed) μ_i : Evolution

Control of Evolving Networks



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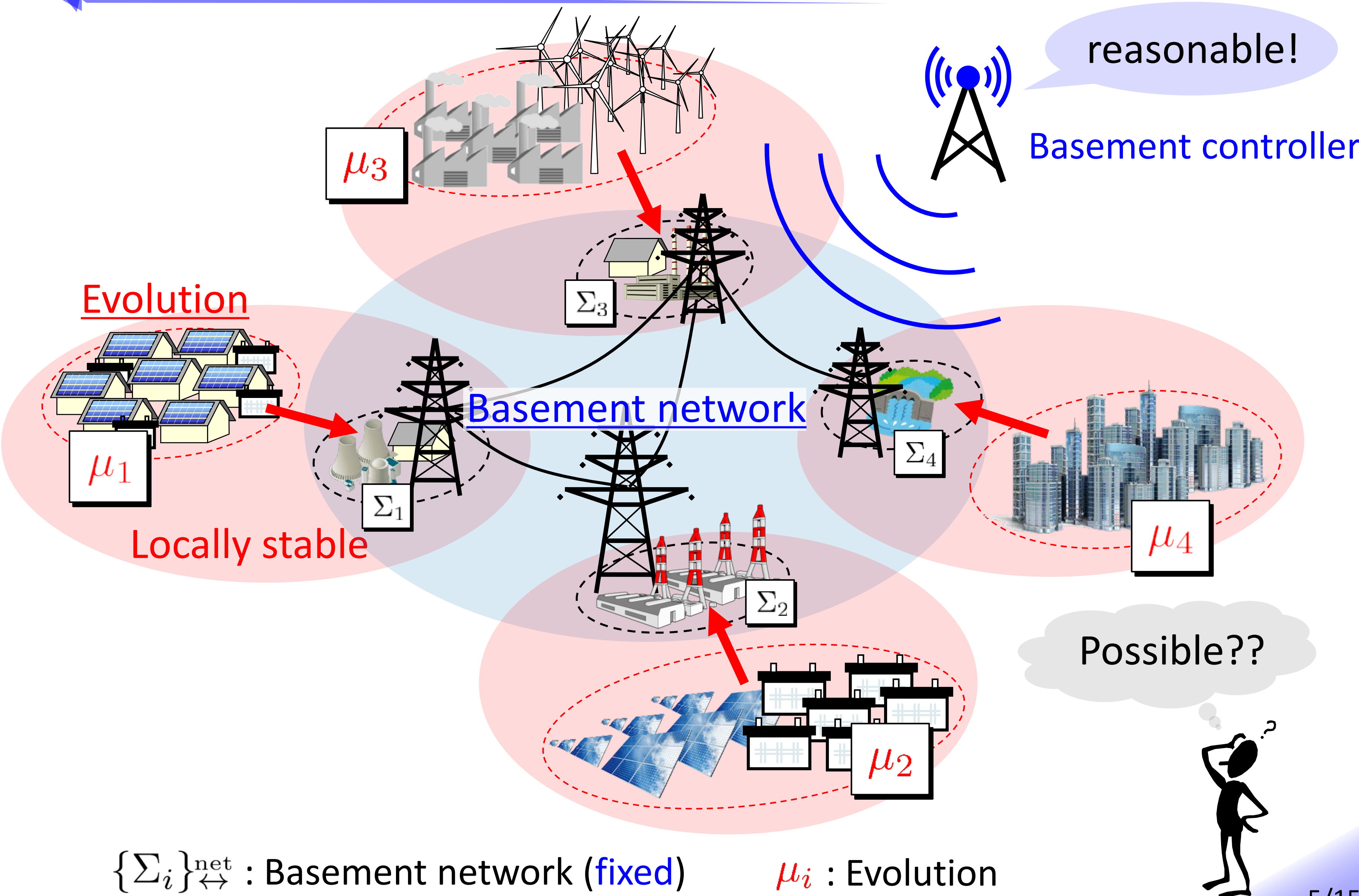
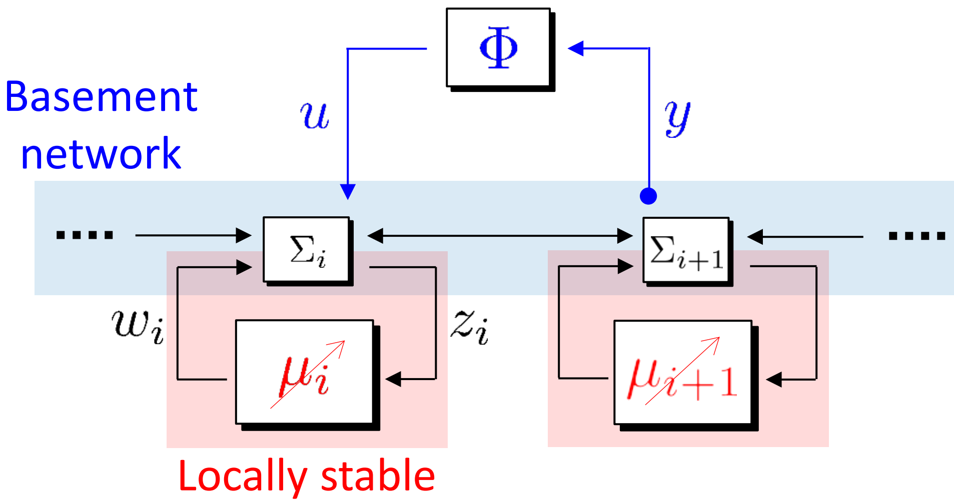
Control of Evolving Networks



$\{\Sigma_i\}^{\rm net}_{\leftrightarrow}$: Basement network (fixed) μ_i : Evolution

Problem Formulation



Disconnected local closed-loop

$$\Sigma_i : \begin{cases} \dot{x}_i = A_i x_i + R_i w_i \\ z_i = S_i x_i \end{cases}$$

$\mu_i : w_i = \mathcal{F}_i(z_i)$ Locally stable

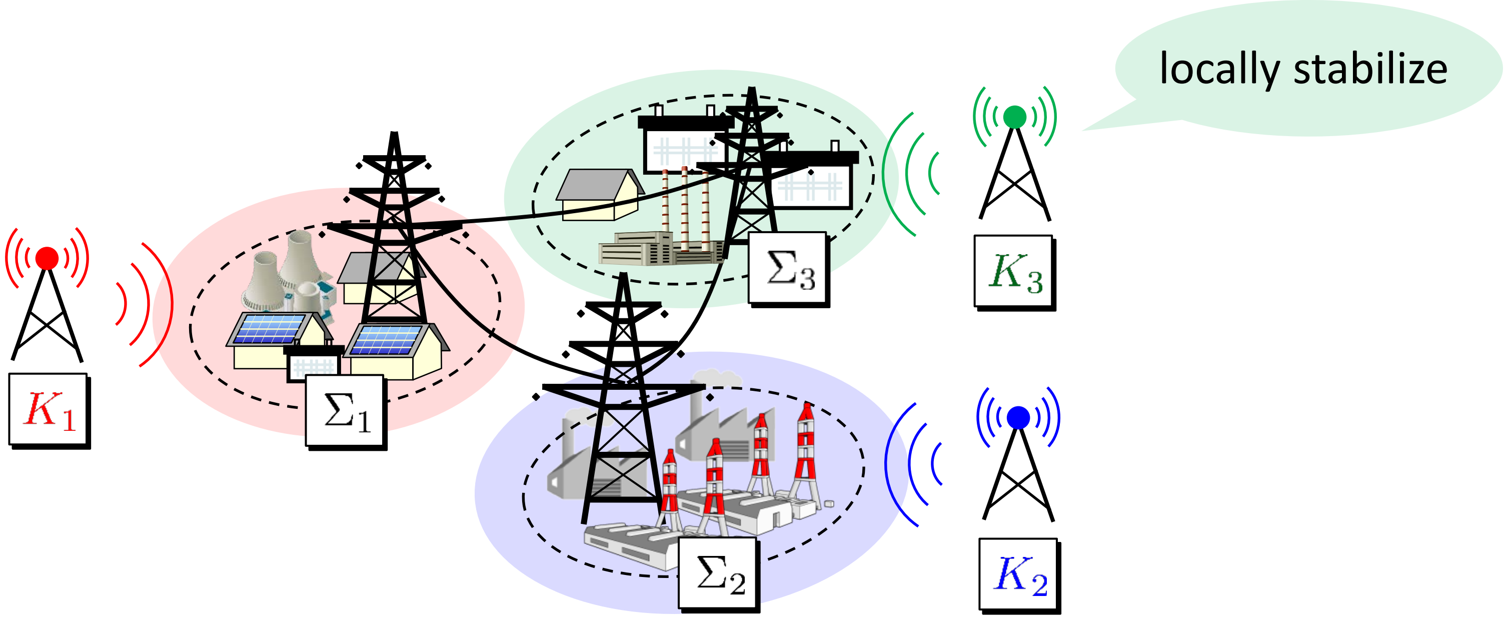
Evolution (nonlinear dynamical map)

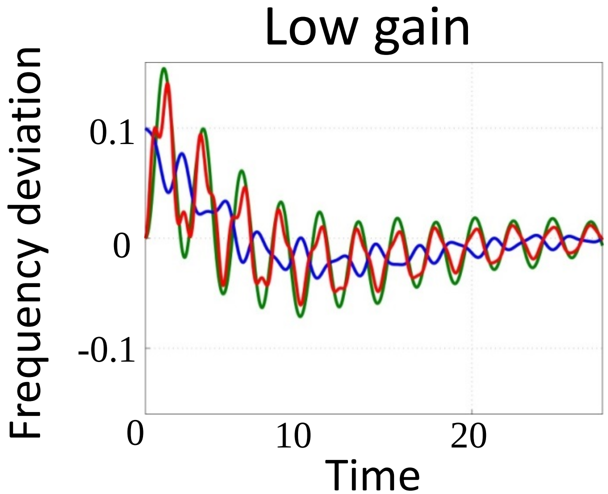
$$\{\Sigma_i\}_{\leftrightarrow}^{\mathrm{net}} : \begin{cases} \dot{x} = (\mathrm{diag}(A_i) + \Gamma)x + Bu + \mathrm{diag}(R_i)w \\ y = Cx \\ z = \mathrm{diag}(S_i)x \end{cases}$$

$$\Phi : \begin{cases} \dot{\xi} = K\xi + Ly \\ u = F\xi \end{cases}$$

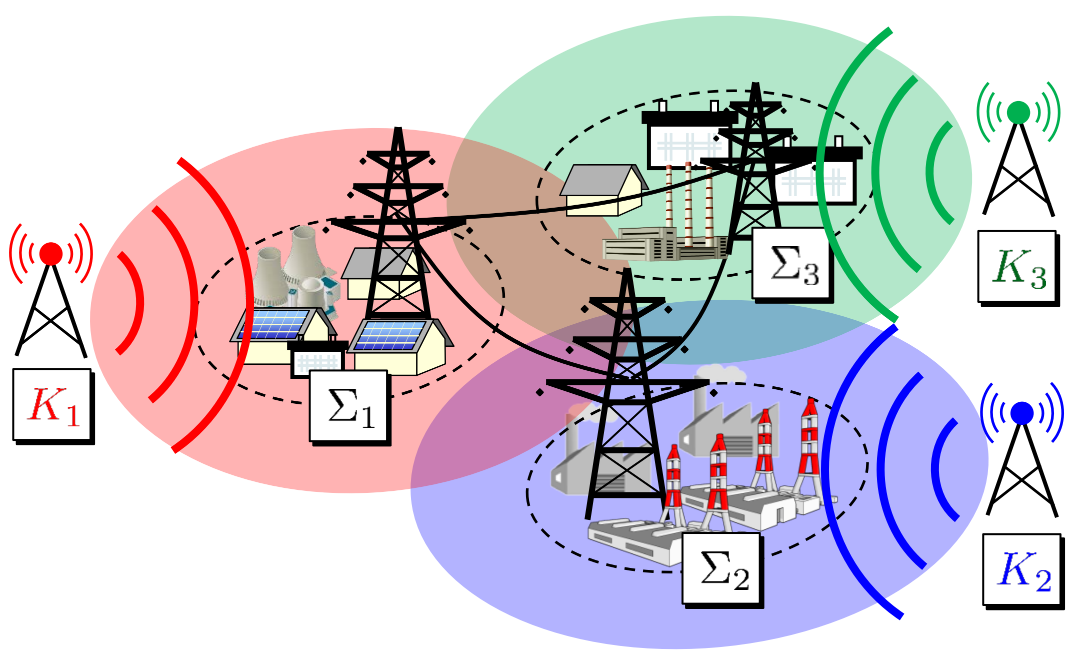
[Problem] Find Φ stabilizing the entire system $(\Phi; \{(\Sigma_i, \mu_i)\}_{\leftrightarrow}^{\mathrm{net}})$ for any $\mu_1, \ldots, \mu_N$ such that each of $(\Sigma_1, \mu_1), \ldots, (\Sigma_N, \mu_N)$ is stable

Difficulty from a Viewpoint of Distributed Control



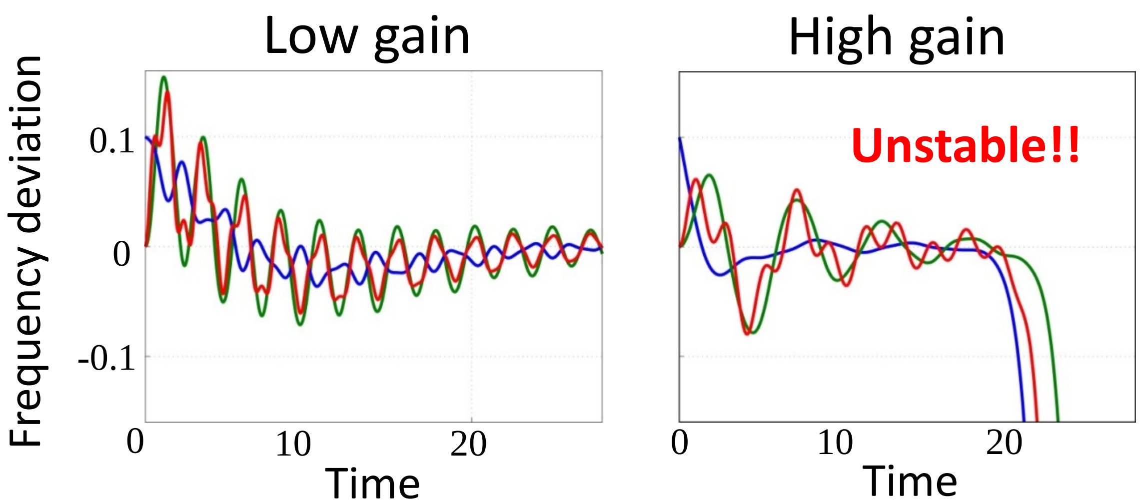


Difficulty from a Viewpoint of Distributed Control



locally stabilize!!

Instability due to the **interference**



Insight from Superposition Principle

Redundant state-space realization

✓ $A = \mathrm{diag}(A_i) + \Gamma$

n-dim. (Basement network)

$$\dot{x} = Ax + Bu + \mathrm{diag}(R_i)w \quad \Leftrightarrow$$

2n-dim.

observing interference

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_i) \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \begin{bmatrix} Bu \\ \mathrm{diag}(R_i)w \end{bmatrix}$$

disconnected subsystems

$x \equiv \xi + \eta$ for any inputs w, u if $x(0) = \xi(0) + \eta(0)$

$x \to 0$ when $\xi \to 0$ and $\eta \to 0$ individually!

Basement control

$u = F\xi$

Locally stable evolution

$\mu_i : w_i = \mathcal{F}_i(z_i)$ with $z_i = S_i \eta_i$

Stabilization of Evolving Networks

[Theorem] If $A + BF$ is stable and $LC = \Gamma$ holds, then $(\Phi; \{(\Sigma_i, \mu_i)\}^{\mathrm{net}}_{\leftrightarrow})$ is stable for any locally stable evolution μ_i .

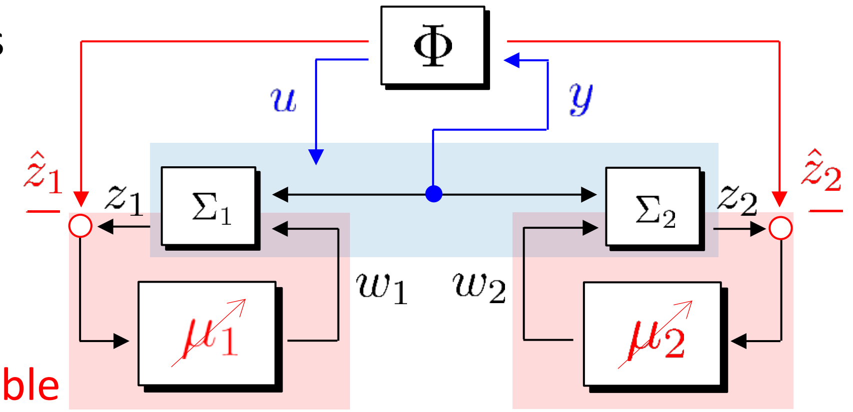
$$\Phi : \begin{cases} \dot{\xi} = \mathrm{diag}(A_i)\xi + Bu + Ly \\ u = F\xi \\ \hat{z} = \mathrm{diag}(S_i)\xi \end{cases} \qquad \{\Sigma_i\}^{\mathrm{net}}_{\leftrightarrow} : \begin{cases} \dot{x} = Ax + Bu + \mathrm{diag}(R_i)w \\ y = Cx \\ z = \mathrm{diag}(S_i)x \end{cases}$$

additional adjustment

$$\mu_i : w_i = \mathcal{F}_i(z_i - \hat{z}_i)$$

✓ Availability of subsystem interaction outputs Γx where $\Gamma = A - \mathrm{diag}(A_i)$

ex) two subsystems

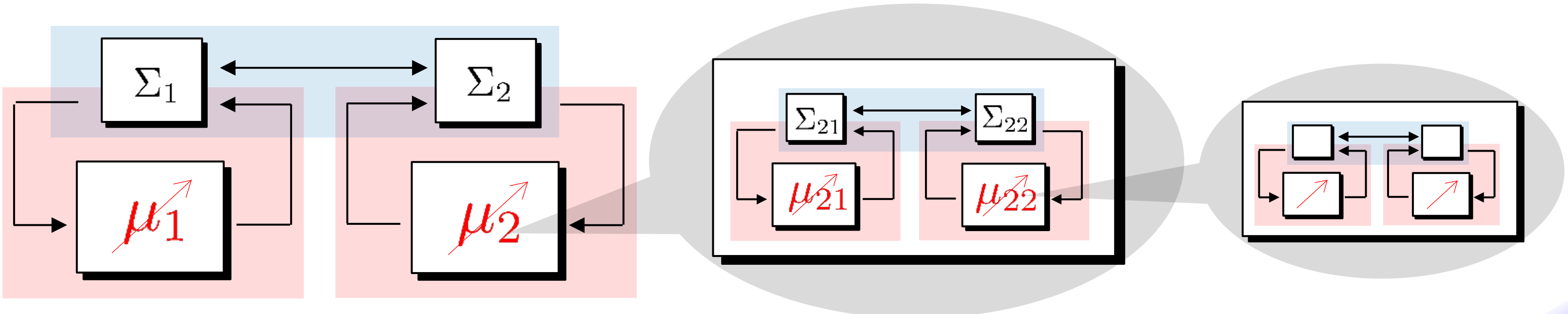


Remarks

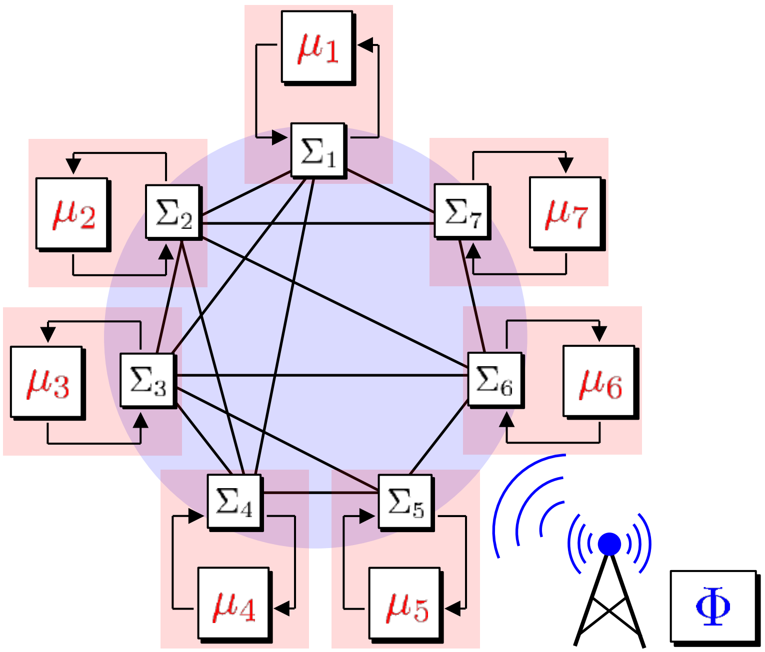
- Condition $LC = \Gamma$ can be relaxed by dynamical observer
  - 2n-dim basement controller Φ (for n-dim basement network)
- Control performance can also be regulated
  - e.g. $\|x\|_{\mathcal{L}_p} \le \|\xi\|_{\mathcal{L}_p} + \|\eta\|_{\mathcal{L}_p}$

$$\dot{x} = Ax + Bu + \mathrm{diag}(R_i)w \iff \begin{cases} \begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_i) \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \begin{bmatrix} Bu \\ \mathrm{diag}(R_i)w \end{bmatrix} \\ x = \xi + \eta \end{cases}$$

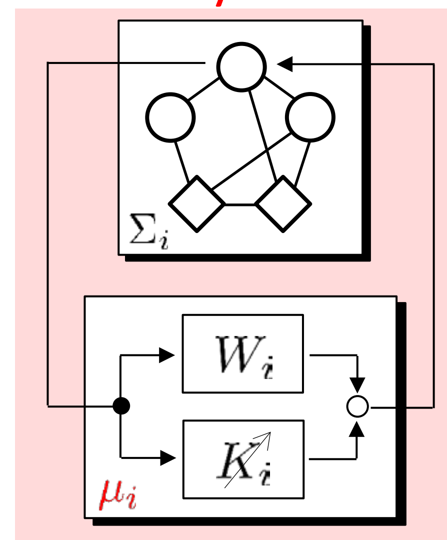
- Scalable generalization via hierarchical implementation



Numerical Example of a Power Network



Locally stable

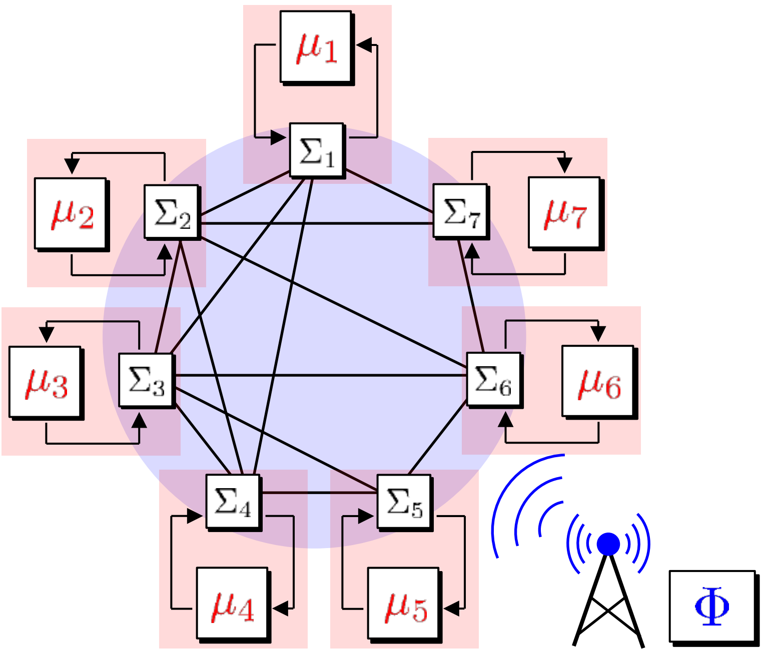


Σ_i : 16-dim
μ_i : 18-dim

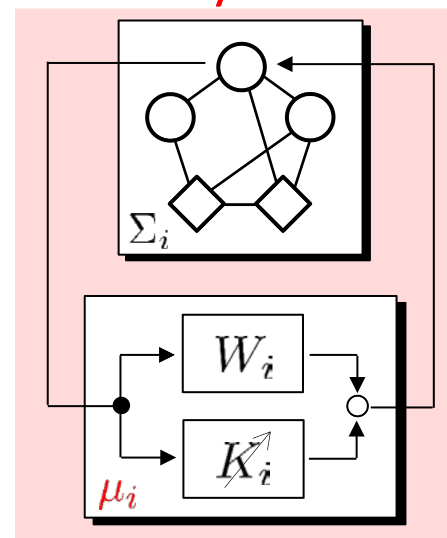
$\{\Sigma_i\}^{\mathrm{net}}_{\leftrightarrow}$: 112-dim + $\{\mu_i\}_{i\in\mathbb{N}}$: 136-dim

- ○ : generator (4-dim = 2-dim swing eq + 2-dim turbine & governor)
- ◇ : load (2-dim swing eq)
- W_i : wind power generator (2-dim swing eq with **4th order nonlinear term**)
- K_i : local controller (18-dim)

Numerical Example of a Power Network



Locally stable



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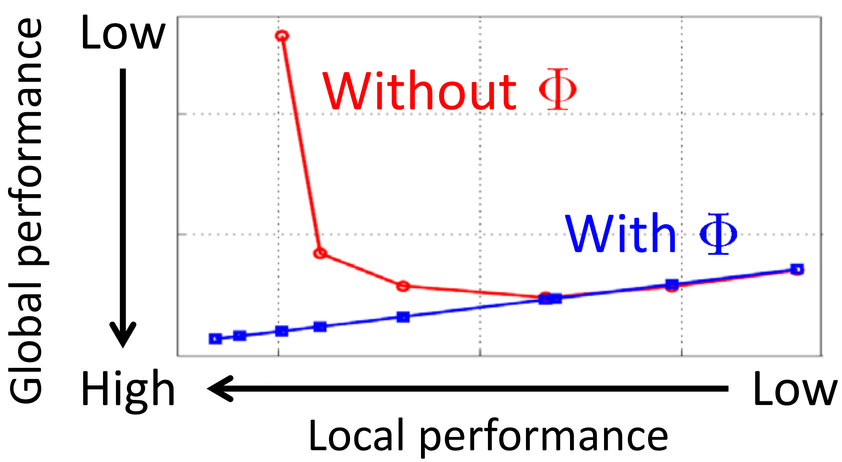

Performance index: $\sup_{\Delta\omega^{\text{Load}}} \dfrac{\|\Delta\omega^{\text{Gen}}\|_{\mathcal{L}_2}}{\|\Delta\omega^{\text{Load}}\|_{\mathcal{L}_2}}$

(degree of frequency deviation stabilization)

<u>Monotonicity wrt GLocal performance</u>

非負システムへの応用の検討

$$\Sigma : \dot{x} = Ax + Bu + \mathrm{diag}(B_i)w$$

✓ $A = \mathrm{diag}(A_i) + \Gamma$

$$\Leftrightarrow \quad \tilde{\Sigma} : \begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_i) \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} + \begin{bmatrix} Bu \\ \mathrm{diag}(B_i)w \end{bmatrix}$$

任意の入力 w, u に対して $x \equiv \xi + \eta$

Σ が非負システムなら？

- $\tilde{\Sigma}$ は非負システム　　動的非負コントローラ設計
- $\|x\|_{\mathcal{L}_1} = \|\xi\|_{\mathcal{L}_1} + \|\eta\|_{\mathcal{L}_1}$　　保守性なく性能評価可能
- $A + \mathrm{diag}(B_i F_i)$ が安定である F_i に対して

$$\Phi : \begin{cases} \dot{\xi} = \mathrm{diag}(A_i + B_i F_i)\xi + \Gamma x \\ \hat{z} = \mathrm{diag}(S_i)\xi \end{cases}$$

分散化可能

まとめ

- ネットワークシステムに対する階層分散制御
  - 不変な基盤ネットワーク ＋ 変化（発展）するサブシステム
  - 階層的な局所安定性によりシステム全体の安定性を保証

- 非負ネットワークシステムへの応用
  - 動的非負コントローラの設計
  - 局所的な制御性能と大局的な制御性能の単調性
  - 分散状態フィードバックの設計により動的コントローラの分散化