2nd ESIC-HARPS Workshop on Smart Grid and Renewable Energy

Power Profile Markets: Power Profile Balancing under Large-Scale Penetration of Battery Storage

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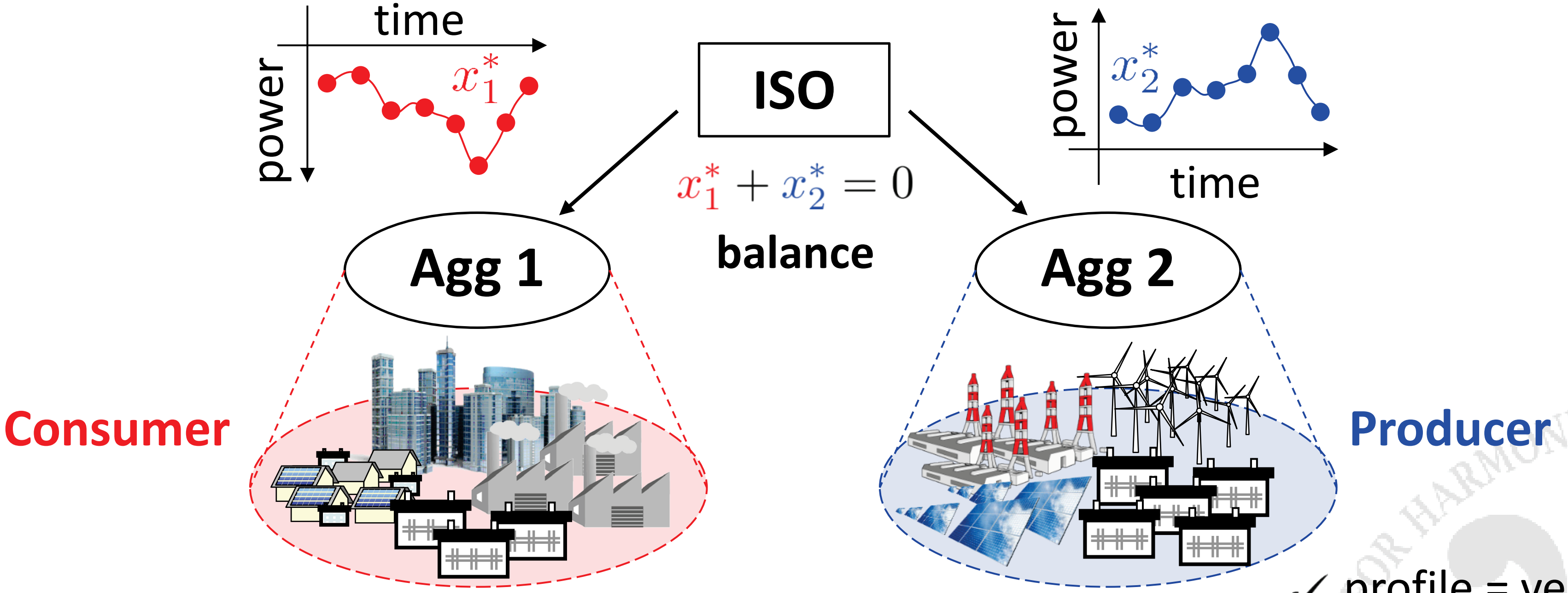
Introduction

Large-scale penetration of battery storage

- improve dispatchability of renewables
- **power profile control** is necessary to regulate unpredicted battery use



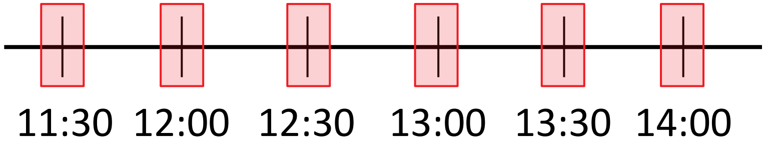
✓ profile = vector

Power profile market for competitive aggregators

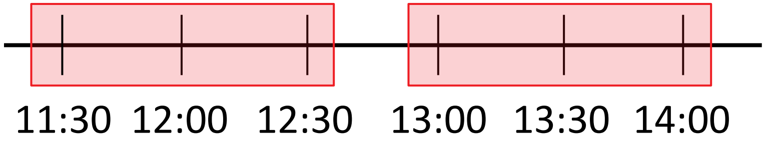
Ways to Market Clearing

for efficient use of battery

Present: time point market clearing

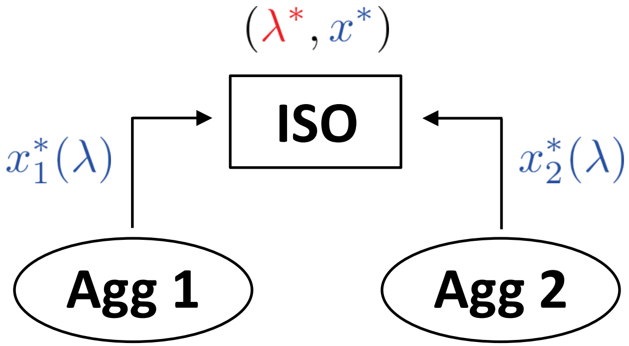


Prospect: time period market clearing



Market mechanism = Math program: $\min_{x} \sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha)$ s.t. $\sum_{\alpha\in\mathcal{A}} x_\alpha = 0$

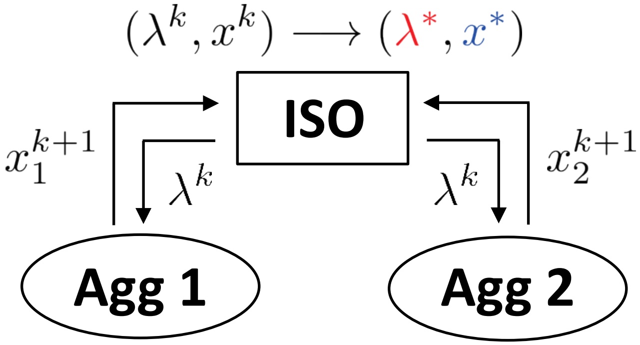
Bidding strategies: offline programs



☺ simple market clearing by bid funs

☹ valid only for scalar-valued products

Dynamical pricing: online iterations



☺ applicable to vector-valued products

☹ premise of strictly convex problem

Goal: Clearing scheme for non-strictly convex power profile markets

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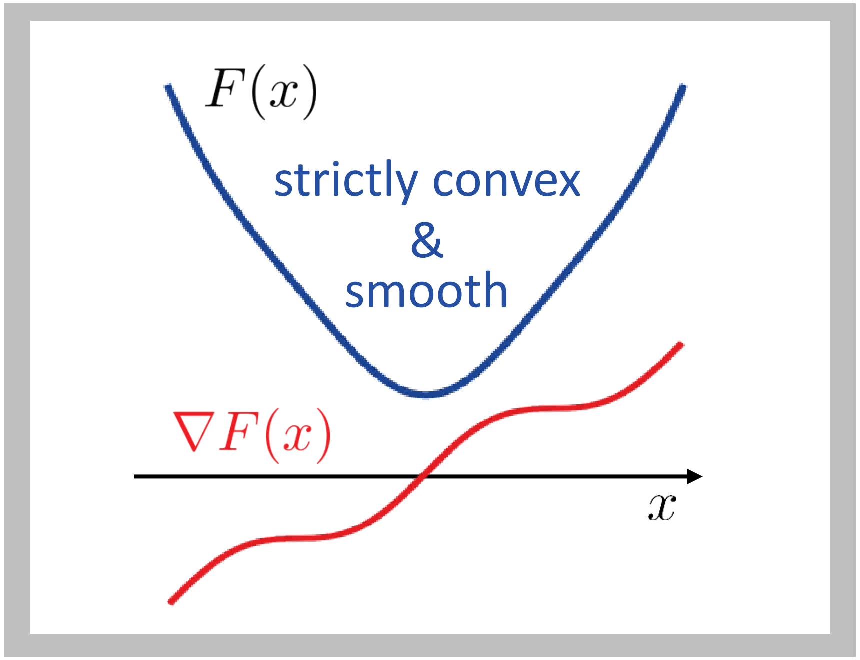
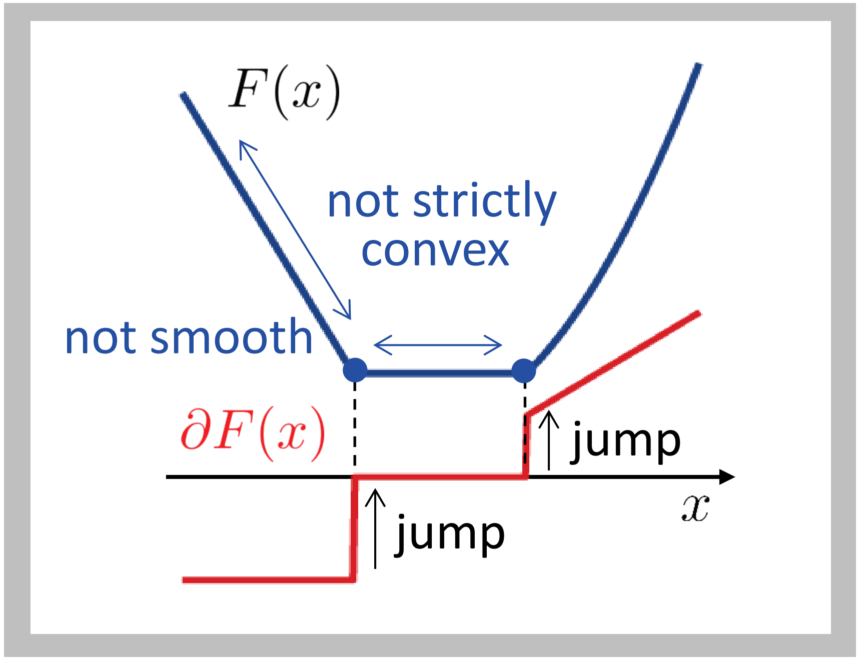
Math Tools from Convex Analysis





Subdifferential: $\partial F(x)$

✓ $\begin{cases}\text{map to set-values if } F \text{ is not smooth} \\ \text{monotone increasing}\end{cases}$

Conjugate transformation:

$$\overline{F}(\lambda) = \sup_x \{\lambda x - F(x)\}$$

$$\begin{tikzcd} \boxed{F(x)} \arrow[r, leftrightarrow, "\text{conjugate}"'] \arrow[d, leftrightarrow] & \boxed{\overline{F}(\lambda)} \arrow[d, leftrightarrow] \\ \boxed{\partial F(x)} \arrow[r, leftrightarrow, "\text{inverse}"] & \boxed{\partial \overline{F}(\lambda)} \end{tikzcd}$$

$\lambda \in \partial F(x)$

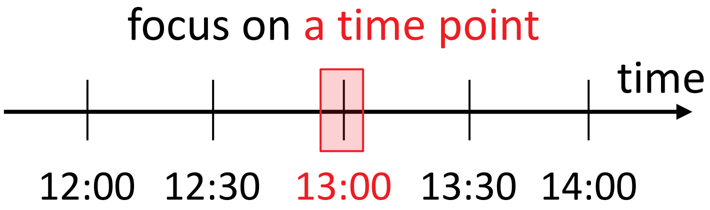
$x \in \partial \overline{F}(\lambda)$

Links to Market Mechanisms

Profit function: $J_\alpha(x_\alpha;\lambda) = \lambda x_\alpha - F_\alpha(x_\alpha)$

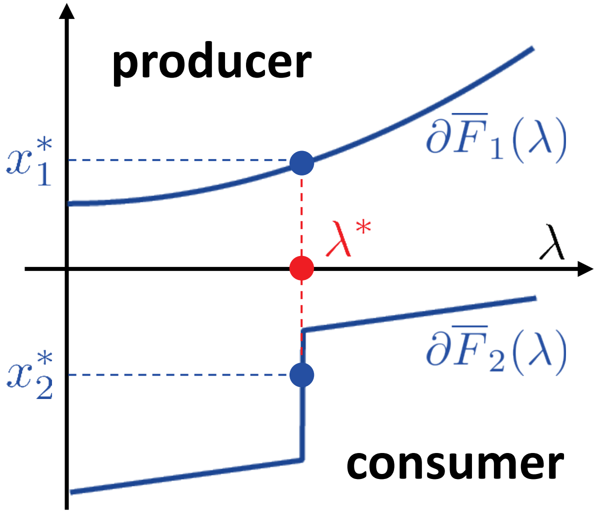
income cost

✓ both x_α and λ are scalars



Maximum profit: $\overline{F}_\alpha(\lambda) = \max_{x_\alpha} J_\alpha(x_\alpha;\lambda)$ **Maximizer:** $x_\alpha^\star \in \partial\overline{F}_\alpha(\lambda)$

bid function



$$x_\alpha^* \in \partial\overline{F}_\alpha(\lambda^*) \ \text{ s.t. } \ \textstyle\sum_{\alpha\in\mathcal{A}} x_\alpha^* = 0$$

$$\Longleftrightarrow \ \min_x \textstyle\sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha) \ \text{ s.t. } \ \textstyle\sum_{\alpha\in\mathcal{A}} x_\alpha = 0$$

$$\Longleftrightarrow \ \max_\lambda \min_x \left\{ \textstyle\sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha) - \lambda \sum_{\alpha\in\mathcal{A}} x_\alpha \right\}$$

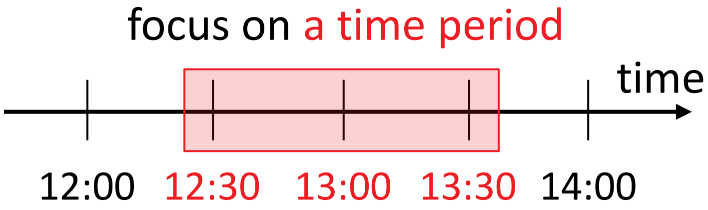
Lagrangian

$$\Longleftrightarrow \ \min_\lambda \max_x \textstyle\sum_{\alpha\in\mathcal{A}} J_\alpha(x_\alpha;\lambda)$$

Bidding Strategy for Profile Markets?

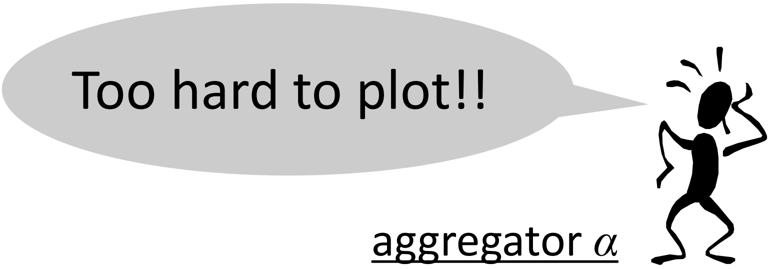
Profit function: $J_\alpha(x_\alpha;\lambda) = \underbrace{\lambda^\mathsf{T} x_\alpha}_{\text{income}} - \underbrace{F_\alpha(x_\alpha)}_{\text{cost}}$

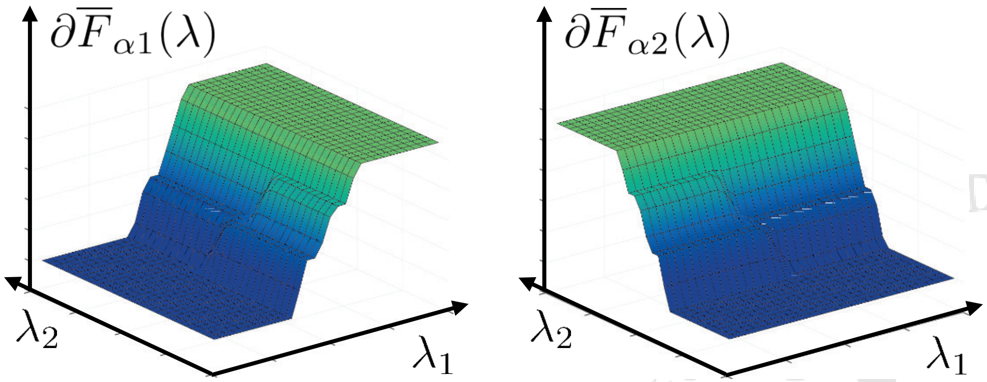
✓ both x_α and λ are **vectors** (profiles)



Market clearing with bid functions: $x_\alpha^* \in \partial\overline{F}_\alpha(\lambda^*)$ s.t. $\sum_{\alpha\in\mathcal{A}} x_\alpha^* = 0$

Bid function: $\partial\overline{F}_\alpha : \mathbb{R}^n \to \mathbb{R}^n$





Bidding strategy is practical only for scalar-valued products

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Cost Function of Power Profiles

Power profile of Aggregator α: $x_\alpha = -l_\alpha + g_\alpha + \eta_\alpha^{\mathrm{out}}\delta_\alpha^{\mathrm{out}} - \frac{1}{\eta_\alpha^{\mathrm{in}}}\delta_\alpha^{\mathrm{in}}$

$$\begin{cases} l_\alpha & \text{: load profile (constant)} \\ g_\alpha \in \mathcal{G}_\alpha & \text{: generation profile} \\ \delta_\alpha = (\delta_\alpha^{\mathrm{in}}, \delta_\alpha^{\mathrm{out}}) \in \mathcal{D}_\alpha & \text{: battery charge \& discharge profiles} \end{cases}$$

✓ profiles in $\mathbb{R}^n$

🟥 : constraints

Generation cost (convex): $G_\alpha(g_\alpha)$ **Battery usage cost** (convex): $D_\alpha(\delta_\alpha)$

- ✓ including evaluation of **final SOC:** $s_\alpha^{\mathrm{fin}}(\delta_\alpha) = s_\alpha^{\mathrm{ini}} + \mathbf{1}_n^\mathsf{T}(\delta_\alpha^{\mathrm{in}} - \delta_\alpha^{\mathrm{out}})$

【Lemma】 $F_\alpha(x_\alpha) = \min_{(g_\alpha, \delta_\alpha) \in \mathcal{F}_\alpha(x_\alpha)} \left\{ G_\alpha(g_\alpha) + D_\alpha(\delta_\alpha) \right\}$ is convex.

- ✓ **uncertain renewables** can be involved as robust/stochastic optimization

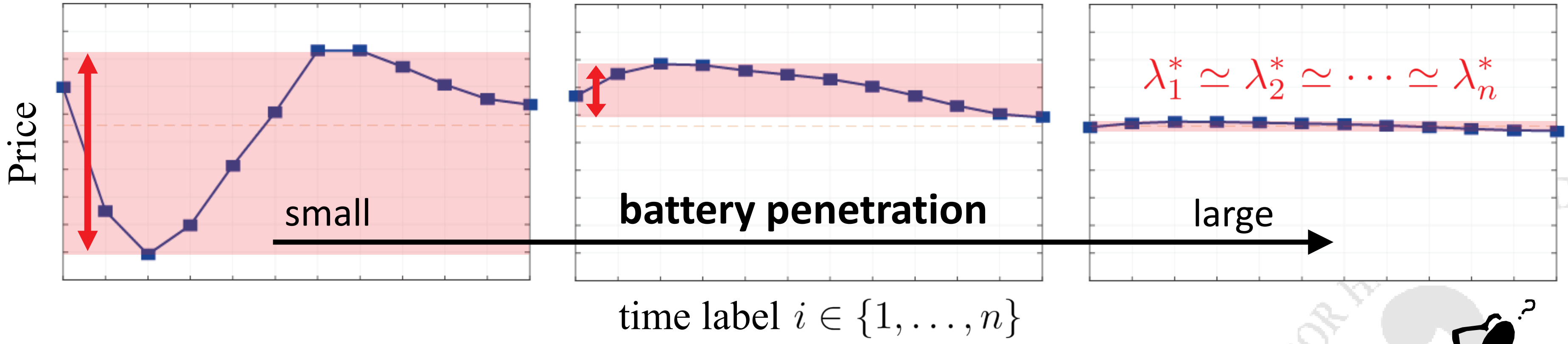
Conjecture from Numerical Simulations

Producer: $x_1 = g_1$ $\qquad$ $F_1(x_1) = G_1(x_1)$

Consumer: $x_2 = -l_2 + \eta^{\mathrm{out}} \delta_2^{\mathrm{out}} - \frac{1}{\eta^{\mathrm{in}}} \delta_2^{\mathrm{in}}$ $\qquad$ $F_2(x_2) = \min\limits_{\delta_2 \in \mathcal{F}_2(x_2)} D_2(\delta_2)$

Market clearing: $\max\limits_{\lambda} \min\limits_{x} \left\{ \sum_{\alpha \in \mathcal{A}} F_\alpha(x_\alpha) - \lambda^\mathsf{T} \sum_{\alpha \in \mathcal{A}} x_\alpha \right\}$ Lagrangian

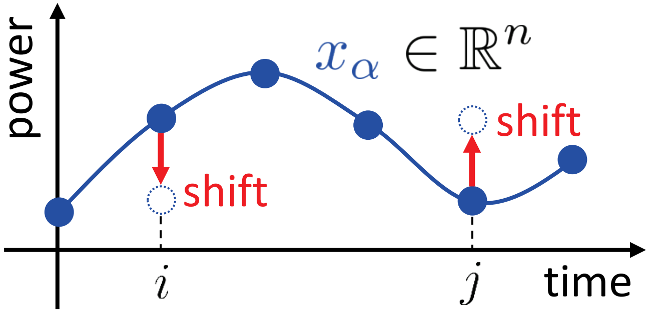
Clearing price $\lambda^* \in \mathbb{R}^n$ when varying the degree of battery penetration:



Large-scale battery penetration leads to price leveling-off?

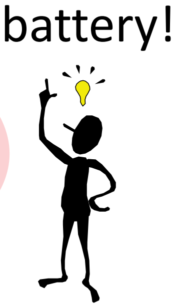
Mathematical Deduction

【Definition】 A power profile x_α is said to be **shiftable** between time points i and j if $\nabla F_\alpha^{\mathsf{T}}(x_\alpha)(e_i - e_j) = 0$.

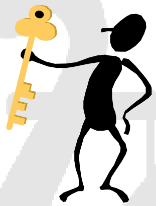


shiftability without changing cost $F_\alpha(x_\alpha)$

non-strict convexity



【Theorem】 There exists some aggregator $\alpha \in \mathcal{A}$ such that $x_\alpha^* \in \partial \overline{F}_\alpha(\lambda^*)$ is shiftable between time points i and j iff $\lambda_i^* = \lambda_j^*$.

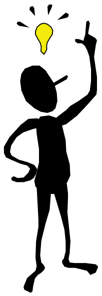
Price levelling-off deduced from battery capacity margin

Pricing via Energy Bid Functions

Power profile profit function: $J_\alpha(x_\alpha;\lambda) = \lambda^\top x_\alpha - F_\alpha(x_\alpha)$

Energy profit function: $J_\alpha(x_\alpha;\mathbf{1}_n\lambda_\mathrm{e}) = \lambda_\mathrm{e}\,\mathbf{1}_n^\top x_\alpha - F_\alpha(x_\alpha)$

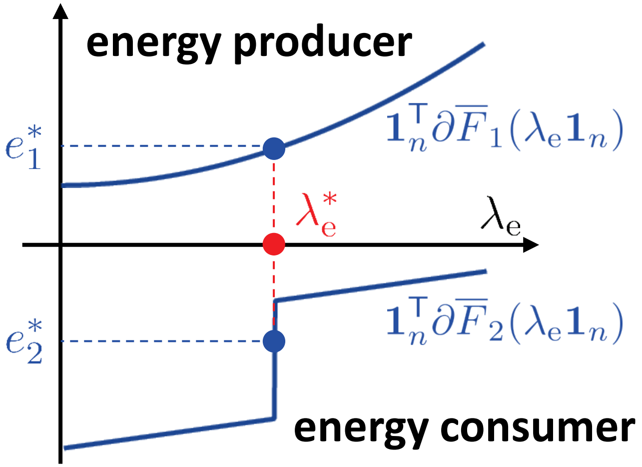
price
levelling-off
$\lambda = \mathbf{1}_n\lambda_\mathrm{e}$

Profit maximum energy: $e_\alpha^\star \in \mathbf{1}_n^\top \partial\overline{F}_\alpha(\mathbf{1}_n\lambda_\mathrm{e})$



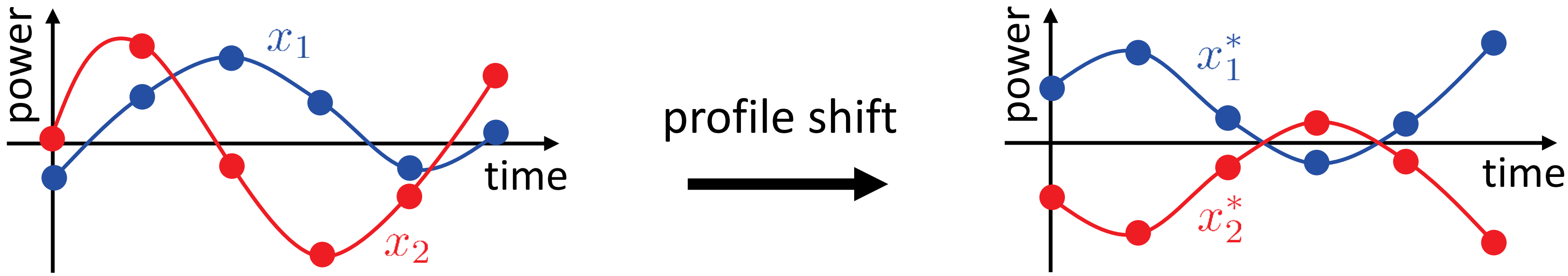
plottable bid function
for energy trade!

✓ bid functions = offline programs



Levelling-off price can be found via offline programs

From Energy Balance to Profile Balance



$e_1^* + e_2^* = 0$ but $x_1 + x_2 \neq 0$

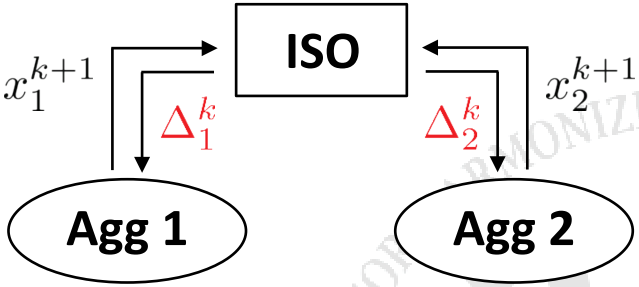
$e_1^* + e_2^* = 0$ and $x_1^* + x_2^* \simeq 0$

Profile imbalance minimization: $\min_x \left\|\sum_{\alpha\in\mathcal{A}} x_\alpha\right\|^2$ s.t. $x_\alpha \in \mathcal{X}_\alpha$

where $\mathcal{X}_\alpha = \{ x_\alpha :$ Energy profit function $J_\alpha(x_\alpha; \mathbf{1}_n\lambda_{\mathrm{e}}^*)$ is maximized $\}$

Decentralized scheme by ADMM:

$$x_\alpha^{k+1} = \mathrm{proj}_{\mathcal{X}_\alpha}\left\{x_\alpha^k - \left(\underbrace{\sum_{i\le\alpha-1} x_i^{k+1} + \sum_{i\ge\alpha} x_i^k}_{\text{imbalance } \Delta_\alpha^k}\right)\right\}$$



【Theorem】 $x_\alpha^k \to x_\alpha^*,\ \forall\alpha\in\mathcal{A}.$ In particular $\sum_{\alpha\in\mathcal{A}} x_\alpha^* = 0$ if $\lambda^* = \mathbf{1}_n\lambda_{\mathrm{e}}^*.$

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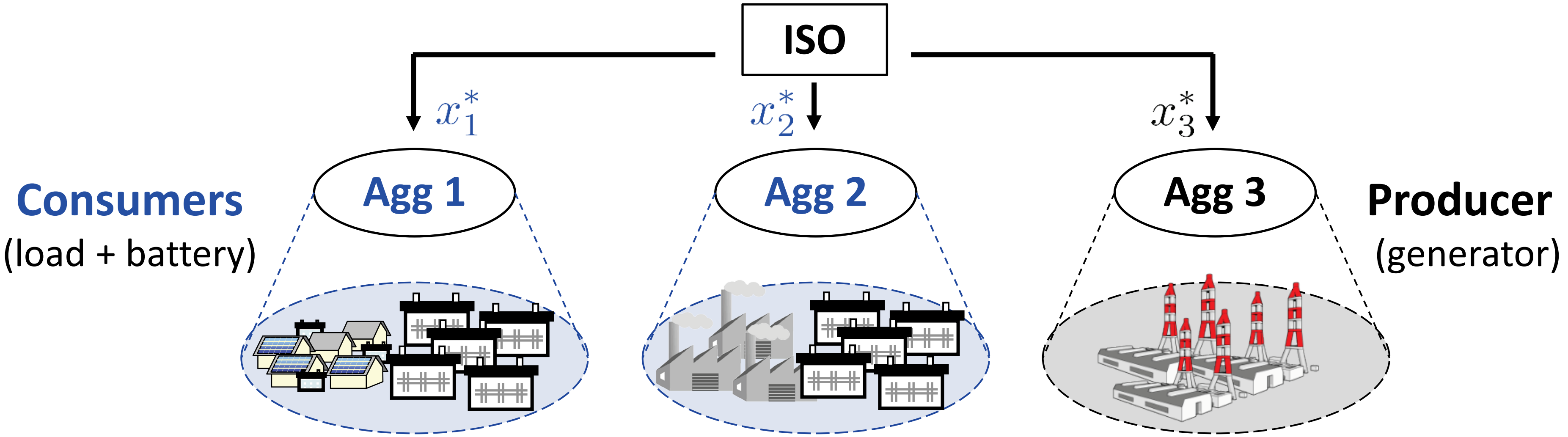
✓ scale is **not yet adjusted** to real situation

Numerical Verification



ISO

x_1^* x_2^* x_3^*

Agg 1 **Agg 2** **Agg 3**

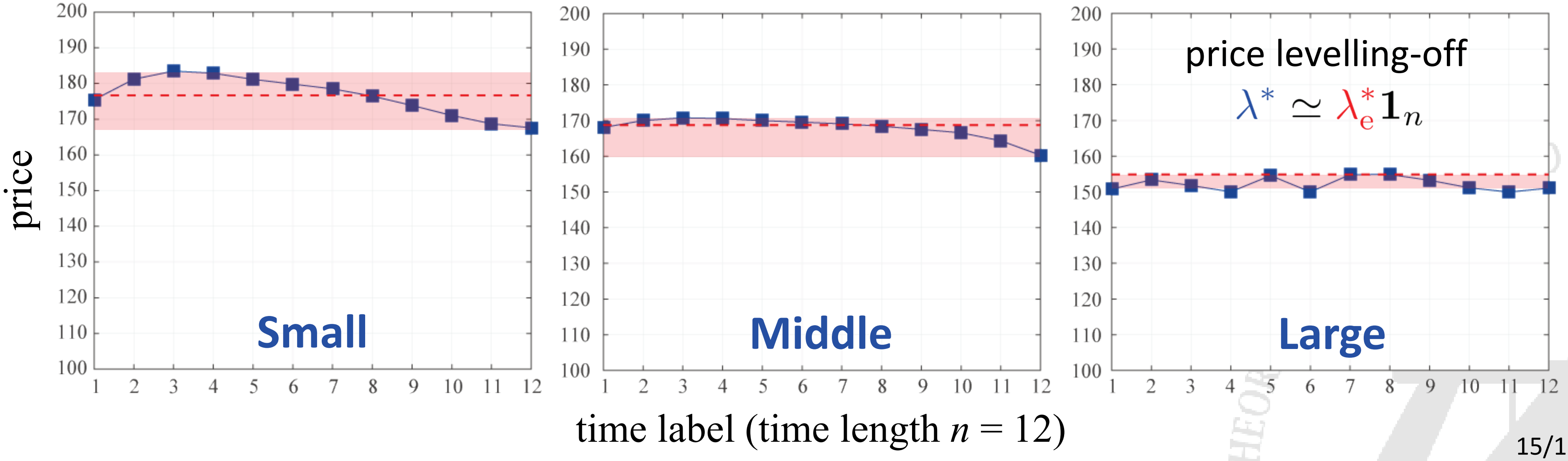
Consumers

(load + battery)

Producer

(generator)

Clearing price $\lambda^* \in \mathbb{R}^n$ when varying the degree of battery penetration:

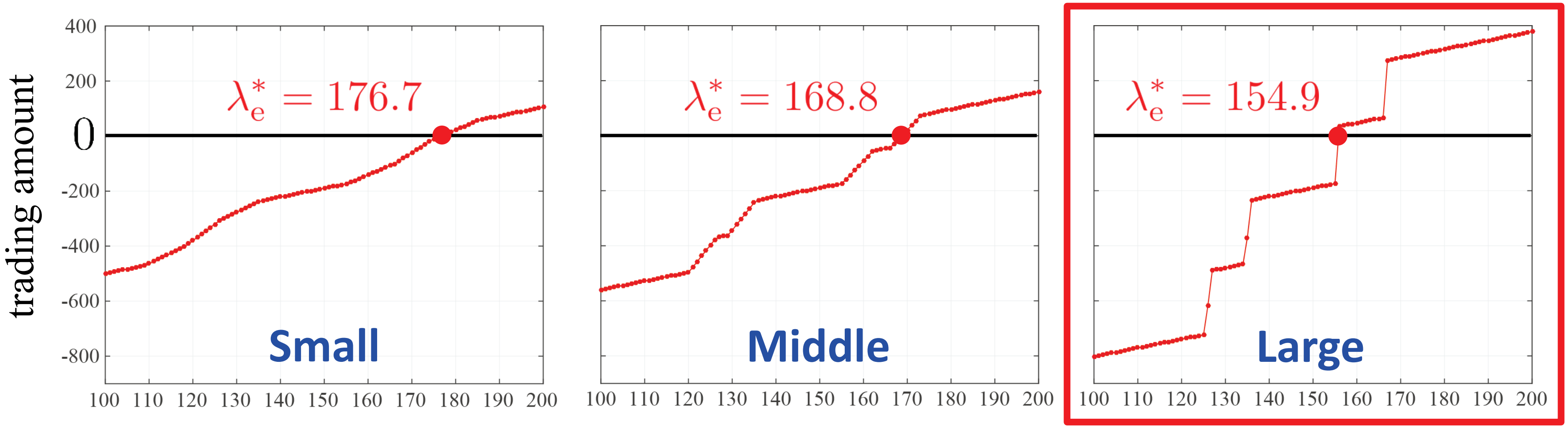


price levelling-off

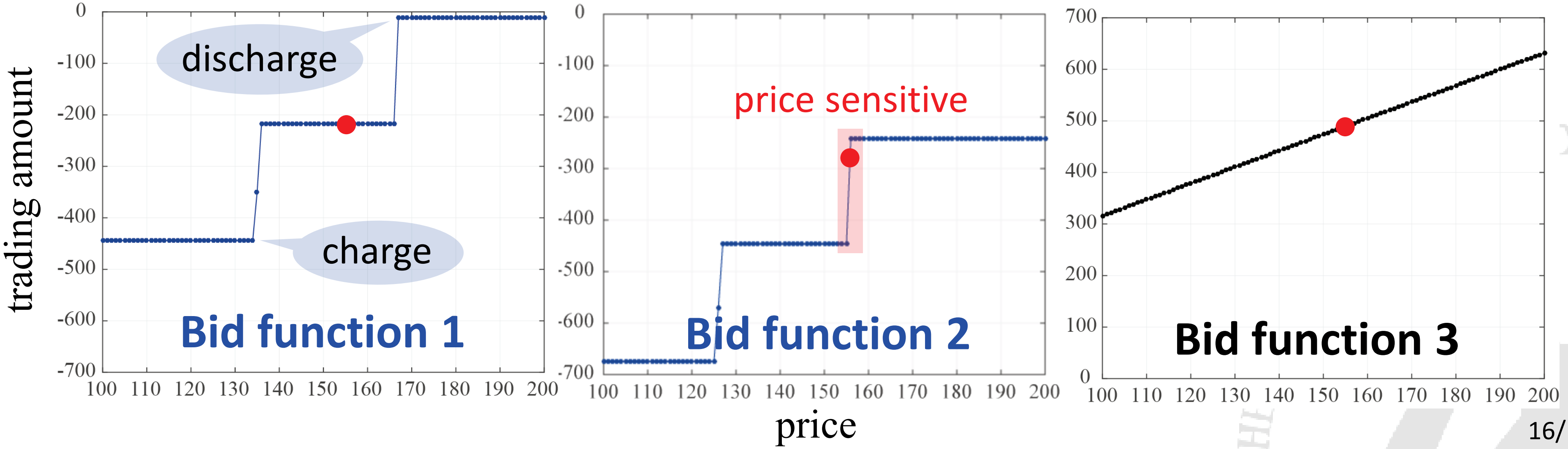
$\lambda^* \simeq \lambda_e^* \mathbf{1}_n$

Pricing by Energy Bid Functions

Total bid functions:

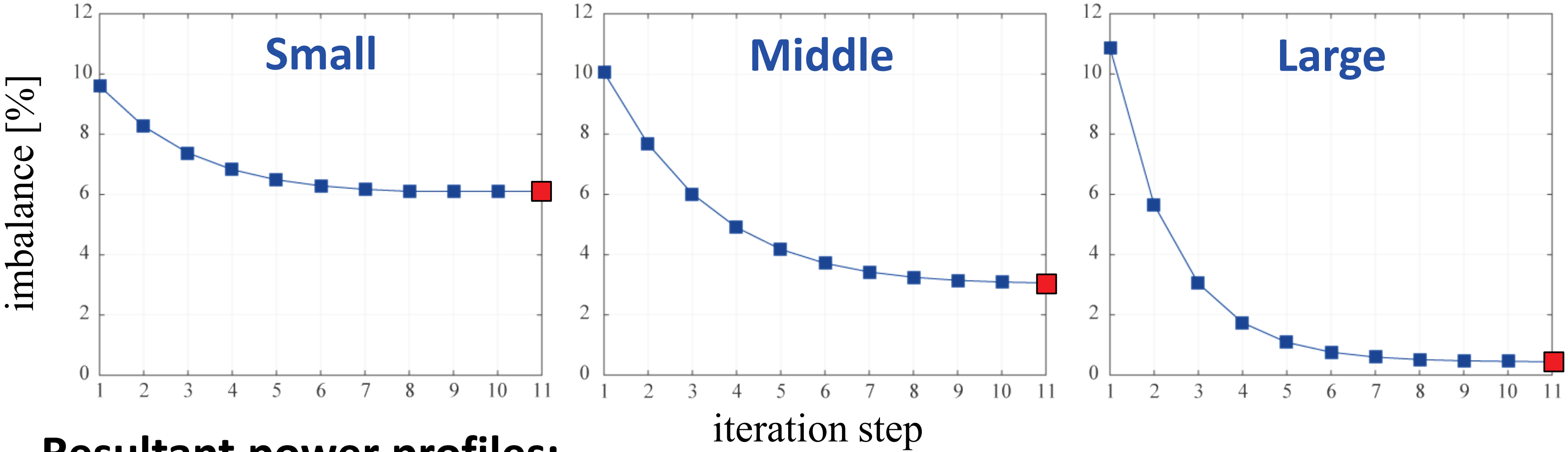


Bid functions in the large battery case:

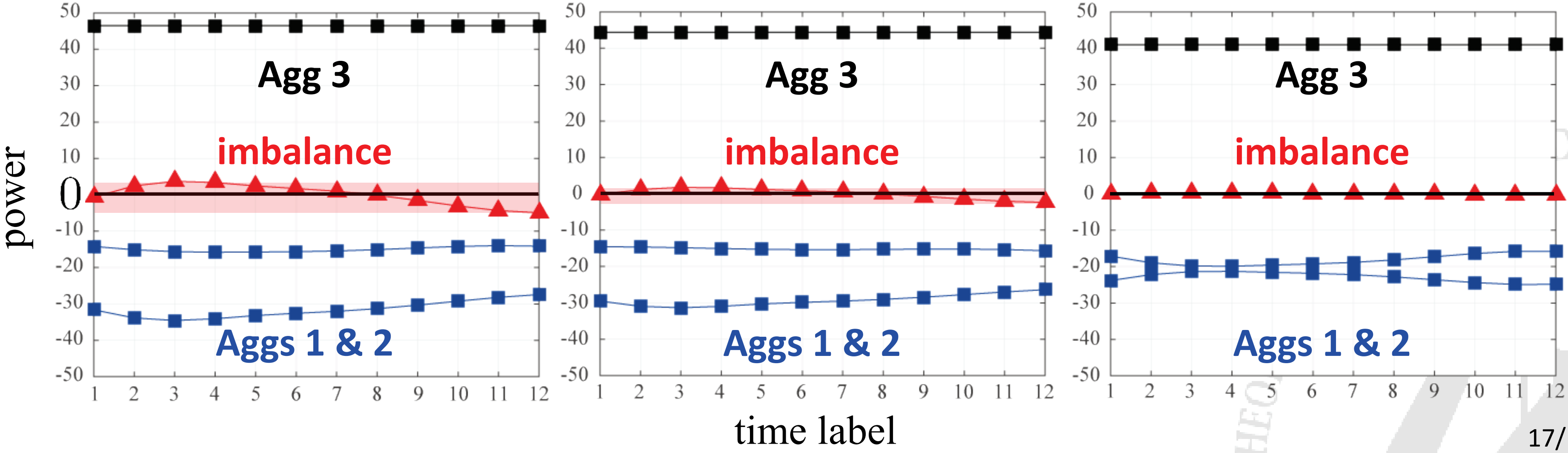


Profile Imbalance Minimization

Profile imbalance minimization by ADMM:



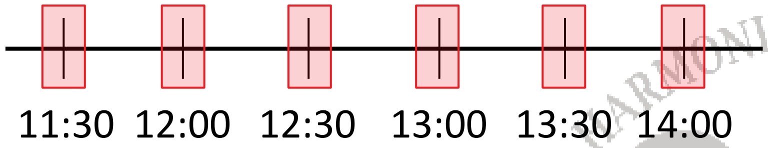
Resultant power profiles:



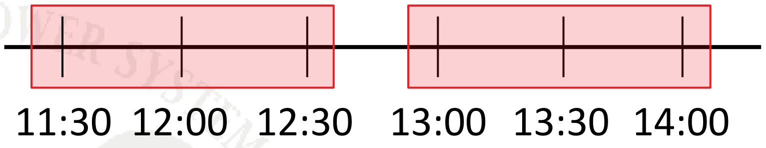
Concluding Remarks

for efficient use of battery

Present: time point market clearing



Prospect: time period market clearing



- **Power profile markets for competitive aggregators**
  - large-scale battery penetration leads to **price levelling-off**
    - profile shiftability owing to battery capacity margin
    - non-strictly convex programs
  - levelling-off price is found by **energy bid functions** (offline programs)
  - **decentralized profile imbalance minimization** by ADMM
- **Future works**
  - analysis on the best length of time periods

Thank you for your attention!