Bidding System Design for Multiperiod Electricity Markets: Pricing of Stored Energy Shiftability

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***Abstract*— In this paper, we design a bidding system for a multiperiod electricity market in which market players participate with power generators and energy storage resources. For the bidding system design, we first develop a sequential procedure to determine a separate multidimensional bid function, i.e., an ensemble of period-specific bid functions, which enables to regard the multiperiod electricity market as an ensemble of conventional period-specific electricity markets. This sequential determination also enables to construct a distributed approximate scheme for multiperiod electricity market clearing. Then, based on a basis transformation similar to the Fourier transformation, we propose a bidding system with explicit consideration of the pricing of energy shiftability. It is shown that, in the situation where the optimal price profile levels off due to high penetration of energy storage, the distributed approximate scheme in the Fourier-like basis can attain the optimal market clearing with the minimal social cost. In addition, we numerically investigate the resultant deadweight loss, i.e., an increase of social costs caused by approximation, varying the levels of energy storage penetration.**

I. INTRODUCTION

The development of a smart grid has been recognized as one of key issues in addressing environmental and social concerns; see [1], [2] for pedagogical overviews. In particular, towards effective integration of dispatchable and renewable power generation, the potential of energy storage has been attracting international attention in smart grid community. Along this trend, developing a *multiperiod electricity market*, as opposed to a conventional period-specific electricity market, is crucial for making use of the power shiftability of energy storage resources and flexible loads. The significance and difficulty of developing such an electricity market has been discussed in the literature; see, e.g., Section 4.3 of [1].

With this background, we have formulated in [3] a multiperiod electricity market as a generalization of a conventional period-specific electricity market. In this formulation, each aggregator aims at making the highest profit by transacting the prosumption (production and consumption) of energy amounts at multiple timeslots, called a *prosumption profile*. The prosumption profile is represented as a vector of prosumption energy amounts. Each aggregator generates a prosumption profile based on available energy resources, such as dispatchable power generation and energy storage.

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Devising a clearing scheme for the multiperiod electricity market is not straightforward due to the multidimensionality (time dependence) of prosumption profiles; see Section III-A for the details. For this issue, we have also proposed in [3] a distributed market clearing scheme implemented as an indirect communication among aggregators through an independent system operator (ISO). This is developed based on the premise that the optimal clearing price profile exactly levels off. Even though we give a theoretical clarification that such exact price levelling off is expected when the penetration of energy storage is sufficiently high, devising a market clearing scheme without assuming the exact price levelling off is left as an open question there.

As generalization of the previous work, this paper develops a more general bidding system for the multiperiod electricity market that can attain prosumption balancing without assuming the exact price levelling off. To this end, we discuss the following items:

- We propose a sequential procedure to determine an ensemble of conventional period-specific bid functions for the multiperiod electricity market.
- Based on a basis transformation similar to the Fourier transformation, we propose a bidding system that takes explicit account of the pricing of energy shiftability.

Section IV in this paper is devoted to the first item. The proposed sequential procedure can be seen as an approximation method to reduce an additively indecomposable cost function to an additively decomposable function. In general, the cost function of energy storage is not additively decomposable because it is temporally dependent. The additively decomposable approximant leads to an ensemble of period-specific bid functions, which enables to regard the multiperiod electricity market as an ensemble of conventional period-specific electricity markets. This also enables to construct a bidding system implemented as a distributed approximate scheme for market clearing. It should be noted that this bidding system necessarily complies with the constraint of prosumption profile balance even though it may cause a degree of deadweight losses.

Section V is devoted to the second item. The transformed basis, called the *multiresolved basis*, represents an ordered basis compatible with temporal resolution (frequency) of prosumption and price profiles. The corresponding bidding system in this basis is regarded as a situation where a market for the total prosumption amount on the day of interest opens firstly, a market for shifting prosumption amounts between the morning timeslots and the afternoon timeslots opens secondly, and so force. This pricing of energy shiftability is novel compared with the existing works [4]–[7]. In addition, it is clarified that market clearing with the minimal social cost, i.e., no deadweight loss, can be attained under high penetration of energy storage.

The remainder of this paper is organized as follows. We first overview the formulation of the multiperiod electricity market in Section II. Then, in Section III, we formulate a bidding system design problem. The main contributions are provided in Sections IV and V, both of which include illustrative numerical examples. Finally, concluding remarks are provided in Section VI.

Notation: We denote the set of real values by $\mathbb{R}$, the ith column of the identity matrix by e_i, the n-dimensional all-ones vector by $\mathbf{1}_n$, and the direct product of $S_1, \ldots, S_n$ by

$$S_1 \times \cdots \times S_n = \textstyle\prod_{i \in \{1,\ldots,n\}} S_i.$$

A function $F : \mathbb{R}^n \to \mathbb{R}$ is said to be convex if

$$F\big((1-\lambda)x + \lambda x'\big) \leq (1-\lambda)F(x) + \lambda F(x') \tag{1}$$

for all $\lambda \in (0,1)$ and for every pair of x and x' in the domain such that the value of F is finite. In particular, F is said to be strictly convex if (1) holds with the strict inequality unless $x = x'$. A set-valued function $\boldsymbol{f} : \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone increasing over $\mathcal{X}$ if

$$\big(y - y'\big)^{\mathsf{T}}(x - x') \geq 0, \quad \forall y \in \boldsymbol{f}(x),\ y' \in \boldsymbol{f}(x')$$

for every pair of x and x' in $\mathcal{X}$. An interval domain of $x \in \mathbb{R}^n$ is denoted by $[x] \subset \mathbb{R}^n$.

II. Multiperiod Electricity Markets

A. Aggregator Model

First, we overview the multiperiod electricity market [3] in which several aggregators participate. We provide a mathematical model of one aggregator who provides the prosumption (production and consumption) of energy amounts on the day of interest. Let $\mathcal{T} := \{1, 2, \ldots, n\}$ denote the set of timeslots on the day. The prosumption energy of an aggregator at the tth timeslot can be described as

$$x_t = g_t - l_t + \eta^{\rm out}\delta_t^{\rm out} - \tfrac{1}{\eta^{\rm in}}\delta_t^{\rm in}, \quad t \in \mathcal{T} \tag{2}$$

where $x_t \in \mathbb{R}$ denotes the resultant prosumption energy to the grid, $g_t \in \mathbb{R}_+$ denotes the power generation of dispatchable generators, $l_t \in \mathbb{R}_+$ denotes the load, and $\delta_t^{\rm in} \in \mathbb{R}_+$ and $\delta_t^{\rm out} \in \mathbb{R}_+$ denote the battery charge and discharge power. The positive constants $\eta^{\rm in}$ and $\eta^{\rm out}$ denote the charge and discharge efficiency, respectively, each of which takes a value in $(0, 1]$. Note that the sign of x_t is positive for outflow direction to the grid.

In the following, we denote the stacked vector of a symbol, called a profile, by that without the subscript t. For example, the prosumption profile $x = (x_t)_{t \in \mathcal{T}}$ represents the sequence of prosumption amounts on the day. In the following, the load profile l is supposed to be a given vector. On the other hand, the dispatchable power generation profile g, and the battery charge and discharge power profiles $\delta^{\rm in}$ and $\delta^{\rm out}$ are decision variables. To realize a desired prosumption profile x, the aggregator determines g and $\delta := (\delta^{\rm in}, \delta^{\rm out})$ as complying with the constraints of $g \in \mathcal{G}$ and $\delta \in \mathcal{D}$, which represents the bounds for dispatchable power generation and the limitation of inverter and battery capacities, respectively.

With respect to each prosumption profile x, we denote the feasible subspace of the dispatchable power generation and the battery charge and discharge profiles as

$$\mathcal{F}(x) := \big\{(g, \delta) \in \mathcal{G} \times \mathcal{D} : \text{(2) holds}\big\}. \tag{3}$$

Furthermore, we denote the set of realizable prosumption profiles as

$$\mathcal{X} := \big\{x \in \mathbb{R}^n : \mathcal{F}(x) \neq \emptyset\big\}, \tag{4}$$

which is convex if $\mathcal{G}$ and $\mathcal{D}$ are convex. The cost functions of dispatchable power generation and battery charge and discharge are denoted as

$$G : \mathcal{G} \to \mathbb{R}, \quad D : \mathcal{D} \to \mathbb{R}. \tag{5}$$

As shown in [3], if G and D are both convex over $\mathcal{G}$ and $\mathcal{D}$, respectively, then the prosumption cost function defined by

$$F(x) := \min_{(g,\delta) \in \mathcal{F}(x)} \Big\{G(g) + D(\delta)\Big\} \tag{6}$$

is convex over $\mathcal{X}$. The value of $F(x)$ represents the minimum cost to realize a prosumption profile x. Note that the closed form of F cannot be written down in general. Furthermore, F is generally not strictly convex even though it is necessarily convex if G and D are convex. This is due to the fact that the battery charge and discharge cost function D is not strictly convex because energy storage has the shiftability of energy amounts among timeslots.

B. Derivation of Multidimensional Bid Functions

In this subsection, we discuss a bid function compatible with the multiperiod electricity market. Let $\lambda \in [\lambda]$ denote an n-dimensional price profile for prosumption profile transaction, which corresponds to the sequence of prices on the day of interest. The profit function in transacting a prosumption profile x is defined as

$$J(x; \lambda) := \lambda^{\mathsf{T}} x - F(x), \tag{7}$$

where $\lambda^{\mathsf{T}} x$ represents the total income by transacting the prosumption profile. On the basis of this profit function, the aggregator can determine a set-valued function given as

$$\boldsymbol{x}(\lambda) := \big\{x \in \mathcal{X} : J(x; \lambda) \geq J(x'; \lambda),\ \forall x' \in \mathcal{X}\big\}, \tag{8}$$

which corresponds to the set of x attaining the maximum of the profit function $J(\cdot; \lambda)$ with a fixed price profile λ. As shown in [3], $\boldsymbol{x} : \mathbb{R}^n \to \mathbb{R}^n$ in (8) is monotone increasing if F is convex. Based on this fact, we introduce the following notion of multidimensional bid functions.

Definition 1: A set-valued function $\boldsymbol{x} : \mathbb{R}^n \to \mathbb{R}^n$ is said to be a *multidimensional bid function* with respect to a price profile interval $[\lambda]$ if it is monotone increasing over $[\lambda]$.

The domain and image of a multidimensional bid function are both multidimensional. This reflects the fact that prosumption amount transaction is performed at multiple timeslots. Note that the element with respect to the tth timeslot, i.e., $e_t^\mathsf{T}\boldsymbol{x} : \mathbb{R}^n \to \mathbb{R}$, is not a conventional period-specific bid function, because it is generally a function of the multidimensional price profile. Note that the graph of $e_t^\mathsf{T}\boldsymbol{x}$ is depicted as a multidimensional hyperplane. It is not tractable in the conventional bidding system because one-dimensional bidding curves can only be considered there.

C. Separability of Multidimensional Bid Functions

For multidimensional bid functions, the following separation will be discussed below.

Definition 2: A multidimensional bid function $\boldsymbol{x} : \mathbb{R}^n \to \mathbb{R}^n$ is said to be *separate* over a price profile interval $[\lambda]$ if

$$e_t^\mathsf{T}\boldsymbol{x}(\lambda) = \boldsymbol{x}_t(\lambda_t), \quad \forall t \in \mathcal{T} \tag{9}$$

where $\boldsymbol{x}_t : \mathbb{R} \to \mathbb{R}$ is a bid function with respect to $[\lambda_t]$.

This notion of separation represents the property that each element of a multidimensional bid function is independent from each other. Thus, a separate multidimensional bid function can be regarded as a simple ensemble of conventional period-specific bid functions. In addition, another notion of separation is introduced for convex functions as follows.

Definition 3: A convex function $F : \mathbb{R}^n \to \mathbb{R}$ is said to be *separate* over an interval domain $[x]$ if

$$F(x) = \textstyle\sum_{t\in\mathcal{T}} F_t(x_t) \tag{10}$$

for an ensemble of $F_t : \mathbb{R} \to \mathbb{R}$ being convex over $[x_t]$.

This corresponds to the additive decomposability of cost functions. Definitions 2 and 3 have a clear link as follows.

Lemma 1: Consider an aggregator in Section II-A. Then, the multidimensional bid function $\boldsymbol{x} : \mathbb{R}^n \to \mathbb{R}^n$ in (8) is separate over a price profile interval if and only if the prosumption cost function F in (6) is separate over an interval domain in the realizable prosumption profile set $\mathcal{X}$ in (4).

Proof: We use the facts from convex analysis theory shown in Appendix. The definition of the conjugate implies $\sup_{x\in\mathcal{X}} J(x;\lambda) = \overline{F}(\lambda)$ for J in (7). Furthermore, the first derivative condition for the supremum leads to $\lambda \in \partial F(x)$. Because of the equivalence between the conjugates, we see that $\boldsymbol{x}(\lambda) = \partial\overline{F}(\lambda)$. According to the relation of the subdifferential for separate functions shown in Corollary 31.5.2 of [8], the separation of $\boldsymbol{x}$ over a domain $[\lambda]$ is rephrased as

$$\boldsymbol{x}(\lambda) = \partial\left(\textstyle\sum_{t\in\mathcal{T}} \overline{F}_t(\lambda_t)\right), \quad \lambda \in [\lambda] \tag{11}$$

for an ensemble of $\overline{F}_t : \mathbb{R} \to \mathbb{R}$ being convex over $[\lambda_t]$. This means the separation of $\overline{F}$ over $[\lambda]$, which is equivalent to the separation of F over some $[x]$. This proves the claim. ■

Lemma 1 shows that a multidimensional bid function $\boldsymbol{x}$ is separate over a price interval $[\lambda]$, i.e., the bidding curve can be depicted as a one-dimensional curve, only over a domain $[x]$ where a prosumption cost function F is separate. Note, however, that this assumption of separation does not make sense when an aggregator has energy storage resources, which provides an ability to shift prosumption amounts. From this viewpoint, we see that the clearing of the multiperiod electricity market is not straightforward with a conventional period-specific bidding system. One approach for systematic market clearing is to devise a method to reduce the multidimensional bid function $\boldsymbol{x}$ in (8) to a separate approximant. In Section IV-A below, we will develop such an approximation method for multidimensional bid functions.

III. Problem Formulation

A. Remarks on Multiperiod Electricity Market Clearing

In this section, with slight abuse of notation, we denote a symbol of the αth aggregator by that with the subscript α, e.g., x_α and $\boldsymbol{x}_\alpha$. Furthermore, we denote the tuple of a symbol indexed by $\alpha \in \mathcal{A}$ by that with the subscript $\mathcal{A}$, e.g., $x_\mathcal{A} := (x_\alpha)_{\alpha\in\mathcal{A}}$. As shown in [3], if at least one prosumption cost function F_α is smooth, then there exists the unique price profile, denoted as λ^*, such that

$$\exists x^*_\mathcal{A} \in \textstyle\prod_{\alpha\in\mathcal{A}} \boldsymbol{x}_\alpha(\lambda^*) \ \text{ s.t. } \ \sum_{\alpha\in\mathcal{A}} x^*_\alpha = 0. \tag{12}$$

In the rest of this paper, we call λ^* the optimal clearing price profile and we assume that at least one F_α is smooth for simplicity.

The optimal clearing of the multiperiod electricity market can be rephrased as finding λ^* and $x^*_\mathcal{A}$ that attain the prosumption profile balance in (12). This is equivalent to finding a solution to the convex program of

$$\min_{x_\mathcal{A}} \textstyle\sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha) \ \text{ s.t. } \ \sum_{\alpha\in\mathcal{A}} x_\alpha = 0, \tag{13}$$

whose Lagrange relaxation is given by

$$\max_{\lambda} \min_{x_\mathcal{A}} \textstyle\sum_{\alpha\in\mathcal{A}} \Big\{ F_\alpha(x_\alpha) - \lambda^\mathsf{T} x_\alpha \Big\}. \tag{14}$$

As seen here, the Lagrange multiplier appears as the price profile λ in (7).

Note that the ISO may be able to solve the convex program of (13) or (14) in a centralized manner. However, this is based on the premise that every aggregator submits the full information of the prosumption cost function F_α in (6). Even though this type of centralized optimization is carried out in the PJM market [10], such a centralized scheme is not necessarily desirable from the viewpoint of computational complexity and privacy of competitive aggregators. Therefore, it is indispensable to design an appropriate bidding system for the multiperiod electricity market, regarded as a distributed solution scheme for the convex program.

As another approach, dynamic pricing methods can be found in the literature [5], [7]. This approach is mainly based on the dual ascent algorithm to solve the convex program in (13), or equivalently (14). Even though such an algorithm can be implemented in a distributed manner, the update of each primal variable assumes the strict convexity of the prosumption cost function F_α. In fact, as mentioned in the end of Section II-A, F_α does not generally satisfy the assumption of the *strict* convexity.

B. Bidding System Design Problem

For convenience of discussion below, let us introduce the following terminology.

Definition 4: A tuple of prosumption profiles denoted as $\hat{x}_{\mathcal{A}}$ is said to be *balancing* if $\sum_{\alpha\in\mathcal{A}} \hat{x}_\alpha = 0$.

In the following, we suppose that each aggregator submits the bidding curves of a separate multidimensional bid function, i.e., an ensemble of conventional period-specific bidding curves, to the ISO. Under this supposition, the ISO can determine a clearing price profile as well as a tuple of balancing prosumption profiles. More specifically, let $\hat{\boldsymbol{x}}_\alpha : \mathbb{R}^n \to \mathbb{R}^n$ denote a separate multidimensional bid function of the αth aggregator, which can be regarded as an approximant of the original multidimensional bid function $\boldsymbol{x}_\alpha$ given as in (8). Then, there can be uniquely found an approximate clearing price profile, denoted as $\hat{\lambda}$, such that

$$\hat{x}_{\mathcal{A}} \in \textstyle\prod_{\alpha\in\mathcal{A}} \hat{\boldsymbol{x}}_\alpha(\hat{\lambda}) \tag{15}$$

is balancing if $\sum_{\alpha\in\mathcal{A}} \hat{\boldsymbol{x}}_\alpha$ is strictly monotone increasing.

To evaluate *quality* of a bidding system, we define the following measure for balancing prosumption profiles.

Definition 5: Let $\hat{x}_{\mathcal{A}}$ be a tuple of balancing prosumption profiles. A *deadweight loss* with respect to $\hat{x}_{\mathcal{A}}$ is defined by

$$\Delta(\hat{x}_{\mathcal{A}}) := \textstyle\sum_{\alpha\in\mathcal{A}} F_\alpha(\hat{x}_\alpha) - F^* \tag{16}$$

where F^* denotes the minimum social cost of the convex program in (13).

By definition, we clearly see that the deadweight loss is nonnegative and it becomes zero for the optimal solution of the convex program in (13). On the basis of arguments above, we formulate a bidding system design problem as follows.

Problem: Consider a set of aggregators in Section II-A. Devise a market clearing scheme that can find a tuple of prosumption profiles and a clearing price profile, denoted as $\hat{x}_{\mathcal{A}}$ and $\hat{\lambda}$, such that the following requirements are satisfied.

– Each aggregator submits an ensemble of bidding curves associated with a separate multidimensional bid function $\hat{\boldsymbol{x}}_\alpha$ to the ISO.
– The ISO determines $\hat{x}_{\mathcal{A}}$ and $\hat{\lambda}$ such that $\hat{x}_{\mathcal{A}}$ is balancing and (15) holds.
– The deadweight loss with respect to $\hat{x}_{\mathcal{A}}$ is small enough.

The major difficulty in this problem is how to determine a good separate multidimensional bid function $\hat{\boldsymbol{x}}_\alpha$ such that the resultant deadweight loss is small enough. In the rest of this paper, we will address this issue.

IV. Sequential Determination of Separate Multidimensional Bid Functions

A. Bid Function Determination at Specific Timeslot

In this subsection, we propose a sequential procedure for deriving a separate multidimensional bid function, denoted as $\hat{\boldsymbol{x}}$, which corresponds to an approximant of the original multidimensional bid function $\boldsymbol{x}$ in (8). For simplicity of notation, we drop the subscript α as long as our attention can be focused on one aggregator without distinction of them. With the notation of stacked vectors like

$$x_{i:j} := (x_i, x_{i+1}, \ldots, x_j)^{\mathsf{T}},$$

where the subscript is associated with timeslots, we suppose that a price subprofile $\hat{\lambda}_{1:t-1}$ as well as a transacted prosumption subprofile $\hat{x}_{1:t-1}$ have been determined by the ISO for the first to $(t-1)$th timeslots. Our objective here is to determine a period-specific bid function $\hat{\boldsymbol{x}}_t : \mathbb{R} \to \mathbb{R}$ for the tth timeslot, based on the premise that $\hat{\lambda}_{1:t-1}$ and $\hat{x}_{1:t-1}$ have been determined in advance. Without loss of generality, they are assumed to be empties when $t = 1$.

To formulate a profit function with respect to the tth timeslot, let us consider a cost function in the form of

$$f_t(x_t) := -\min_{\lambda_{t+1:n}} \max_{x_{t+1:n}} \left\{ \lambda_{t+1:n}^{\mathsf{T}} x_{t+1:n} - F(\hat{x}_{1:t-1}, x_{t:n}) \right\} \tag{17}$$

where the minimization and maximization are subject to

$$\lambda_{t+1:n} \in [\hat{\lambda}_{t+1:n}], \quad \sum_{\tau=1}^{t-1} e_\tau \hat{x}_\tau + \sum_{\tau=t}^{n} e_\tau x_\tau \in \mathcal{X}. \tag{18}$$

In this program, the interval $[\hat{\lambda}_{t+1:n}]$, called a price prediction interval, is determined by the aggregator of interest. Over the feasible domain of f_t, denoted by $[\hat{x}_t]$, we consider a period-specific profit function $\hat{J}_t$ defined as

$$\hat{J}_t(x_t; \lambda_t) := \lambda_t x_t - f_t(x_t). \tag{19}$$

Then, we obtain the following period-specific bid function.

Lemma 2: Consider an aggregator in Section II-A and define the sequential profit function $\hat{J}_t$ as in (19). For any price prediction interval $[\hat{\lambda}_{t+1:n}]$ and any constants $\hat{\lambda}_{1:t-1}$ and $\hat{x}_{1:t-1}$, the set-valued function

$$\hat{\boldsymbol{x}}_t(\lambda_t) := \left\{ x_t : \hat{J}_t(x_t; \lambda_t) \geq \hat{J}_t(x'_t; \lambda_t),\ \forall x'_t \in [\hat{x}_t] \right\} \tag{21}$$

is a bid function with respect to any price interval $[\lambda_t]$.

Proof: To prove the claim, we show that $\hat{\boldsymbol{x}}_t$ is monotone increasing. First, we suppose that f_t in (17) is a convex function. Under this supposition, $\hat{\boldsymbol{x}}_t(\lambda_t) = \partial \overline{f}_t(\lambda_t)$ holds where $\overline{f}_t$ denotes the conjugate of f_t. Thus, the monotonicity of the subdifferential proves the monotonicity of $\hat{\boldsymbol{x}}_t$.

Let us show the convexity of f_t. We notice that

$$\overline{F}_t(x_t, \lambda_{t+1:n}) := \max_{x_{t+1:n}} \left\{ \lambda_{t+1:n}^{\mathsf{T}} x_{t+1:n} - F(\hat{x}_{1:t-1}, x_{t:n}) \right\}$$

corresponds to the partial conjugate of the convex function $F(\hat{x}_{1:t-1}, x_t, \cdot)$. As shown in Theorem 33.1 of [8], $\overline{F}_t$ is a saddle function, i.e., it is concave with respect to the first variable and convex with respect to the second. Furthermore, f_t is given as the maximum of the collection of convex functions $-\overline{F}_t(\cdot, \lambda_{t+1:n})$, which is shown to be convex; see Theorem 5.5 of [8]. Thus, f_t is convex. ■

On the basis of the bid function $\hat{\boldsymbol{x}}_t$ in (21), each aggregator can depict own period-specific bidding curve for the tth timeslot market. By definition, provided that the feasible domain $[\hat{x}_t]$ is nonempty, so is $[\hat{x}_{t+1}]$ for any $\hat{x}_t \in \hat{\boldsymbol{x}}_t(\hat{\lambda}_t)$ with a clearing price $\hat{\lambda}_t \in [\lambda_t]$. Thus, according to this sequential determination of $\hat{x}_t$ and $\hat{\lambda}_t$, each aggregator can determine a separate approximant $\hat{\boldsymbol{x}} : \mathbb{R}^n \to \mathbb{R}^n$ as the ensemble of $\hat{\boldsymbol{x}}_t$.

B. Multiperiod Electricity Market Clearing

For a set of aggregators, let $\hat{\boldsymbol{x}}_\alpha : \mathbb{R}^n \to \mathbb{R}^n$ denote the separate multidimensional bid function of the αth aggregator given as in Section IV-A. By aggregating all bidding curves, the ISO can determine a tuple of prosumption profiles and a clearing price profile, denoted as $\hat{x}_{\mathcal{A}}$ and $\hat{\lambda}$, such that $\hat{x}_{\mathcal{A}}$ is balancing and (15) holds. This gives one solution to the bidding system design problem in Section III-B.

Let us discuss the resultant deadweight loss, i.e., the quality of the bidding system. In fact, we can guarantee the optimality at least when every multidimensional bid function $\boldsymbol{x}_\alpha$ is originally separate. This is formally stated as follows.

Theorem 1: Consider a set of aggregators in Section II-A and let $\hat{\boldsymbol{x}}_\alpha : \mathbb{R}^n \to \mathbb{R}^n$ denote the separate multidimensional bid function of the αth aggregator given as in (21). Assume that every multidimensional bid function $\boldsymbol{x}_\alpha$ in (8) is separate over a price profile interval $[\lambda]$. If $\lambda^* \in [\lambda]$ for the optimal price profile λ^* in (12), then the clearing price profile $\hat{\lambda}$ such that $\hat{x}_{\mathcal{A}}$ in (15) is balancing is identical to λ^*. Furthermore, the deadweight loss with respect to $\hat{x}_{\mathcal{A}}$ is zero.

Proof: Let us focus our attention on one aggregator and we drop the subscript α without distinction of aggregators. From the assumption that $\boldsymbol{x}$ is separate over $[\lambda]$, we see that F is separate over some domain $[x]$ as shown in Lemma 1. When F is represented as in (10), f_t in (17) is identical to $\underline{F_t}$ up to an additive constant. Thus, $\hat{\boldsymbol{x}}_t$ in (21) is equal to $\partial\overline{F}_t$. This is also equal to the tth element of $\boldsymbol{x}$ represented as in (11). Therefore, provided that the multidimensional bid functions of all aggregators are separate and $\lambda_t^* \in [\lambda_t]$, the optimal solutions are to be found, namely $\hat{\lambda}_t = \lambda_t^*$ and $\hat{x}_t = x_t^*$, which satisfy $x_t^* \in \partial\overline{F}_t(\lambda_t^*)$. This proves the claim. ■

Theorem 1 can guarantee the optimality of market clearing under the assumption that every $\boldsymbol{x}_\alpha$ is originally separate. However, such an assumption may not make sense especially when an aggregator has large energy storage resources. This will be numerically demonstrated in Section IV-C below.

C. Numerical Example

We consider the multiperiod electricity market consisting of three aggregators. On the day of interest, the transacted prosumption amounts are determined at every 90 minutes, i.e., the dimension of prosumption profiles is $n = 16$. The first and second aggregators are supposed to manage five thousand residential consumers with energy storage resources, formulated as

$$x_\alpha = -l_\alpha + \eta_\alpha^{\mathrm{out}}\delta_\alpha^{\mathrm{out}} - \tfrac{1}{\eta_\alpha^{\mathrm{in}}}\delta_\alpha^{\mathrm{in}}, \quad \alpha \in \{1, 2\}.$$

Furthermore, the third aggregator manages nine generators, formulated as $x_3 = \sum_{i=1}^{9} g_i$, where the subscript i represents the label of generators. In our simulation, we set $\eta_1^{\mathrm{out}} = \eta_2^{\mathrm{in}} = 0.95$ and $\eta_2^{\mathrm{out}} = \eta_2^{\mathrm{in}} = 0.94$. The inverter and battery capacities are supposed to be 3.5 [kW] and 10.5 [kWh],

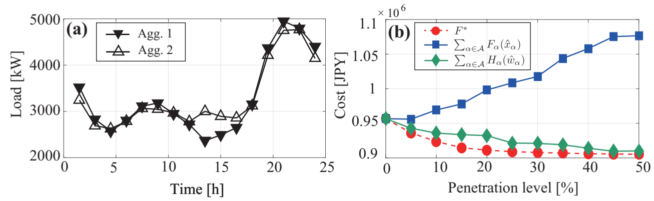

Fig. 1. (a) Load profiles of aggregators. (b) The resultant social costs versus levels of energy storage penetration.

respectively, for one residential consumer. The load profiles l_1 and l_2 are plotted in Fig. 1(a).

The fuel cost function of thermal generation is supposed to be a linear function of

$$G_3(g_1, \ldots, g_9) = \textstyle\sum_{i=1}^{9} 3i \times \mathbf{1}_{16}^{\mathsf{T}} g_i, \tag{22}$$

which is convex but not strictly convex. The lower and upper bounds of g_i are given as

$$g_1, g_2, g_3 \in [0, 1500 \times \mathbf{1}_n], \quad g_4, g_5, g_6 \in [0, 1000 \times \mathbf{1}_n],$$
$$g_7, g_8, g_9 \in [0, 800 \times \mathbf{1}_n].$$

The battery charge and discharge cost functions D_1 and D_2 are given as follows. Let s_α^0 denote the initial amount of stored energy, i.e., the state of charge, which is defined as the deviation from a neutral value. Then, the amount of stored energy deviation at the termination time is represented as

$$s_\alpha(\delta_\alpha) := s_\alpha^0 + \mathbf{1}_n^{\mathsf{T}}(\delta_\alpha^{\mathrm{in}} - \delta_\alpha^{\mathrm{out}}). \tag{23}$$

In our simulation, we set $s_1^0 = s_2^0 = 0$. It is reasonable to suppose that a higher level of final stored energy is more preferable than a lower level, and vice versa. To take into account this aspect, each aggregator is supposed to assess the value of final stored energy by

$$D_\alpha(\delta_\alpha) := -d\left(s_\alpha(\delta_\alpha)\right)$$

where $d : \mathbb{R} \to \mathbb{R}$ is a concave function given as

$$d(s) := \begin{cases} a_4(s - \overline{s}) + a_3\overline{s}, & \overline{s} \le s, \\ a_3 s, & 0 \le s < \overline{s}, \\ a_2 s, & \underline{s} \le s < 0, \\ a_1(s - \underline{s}) + a_2\underline{s}, & s < \underline{s}. \end{cases}$$

We set $a_1 = 20$, $a_2 = 10$, $a_3 = 6.7$, $a_4 = 3.3$, $\underline{s} = -1.31$, and $\overline{s} = 1.31$. In this setting, D_α is also convex but not strictly convex. Then, we obtain a convex prosumption cost function F_α for each aggregator. We give the price prediction interval $[\hat{\lambda}_{2:16}]$ in (18) as the multidimensional interval from 10 to 23 [JPY/kWh].

Varying the levels of energy storage penetration, we calculate the resultant deadweight loss $\Delta(\hat{x}_{\mathcal{A}})$ in (16) when using the separate approximants $\hat{\boldsymbol{x}}_\alpha$ derived as in Section IV-A. In Fig. 1(b), the resultant social costs $\sum_{\alpha\in\mathcal{A}} F_\alpha(\hat{x}_\alpha)$ and F^* are plotted by the blue solid line with squares and the red dotted line with circles, respectively. The difference between them represents the deadweight loss and the horizontal axis represents the percentage of residential consumers having energy storage. From this figure, we see that the deadweight loss is zero at least when there is no energy storage, i.e., the 0% penetration level. This is because $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\boldsymbol{x}_3$ in this

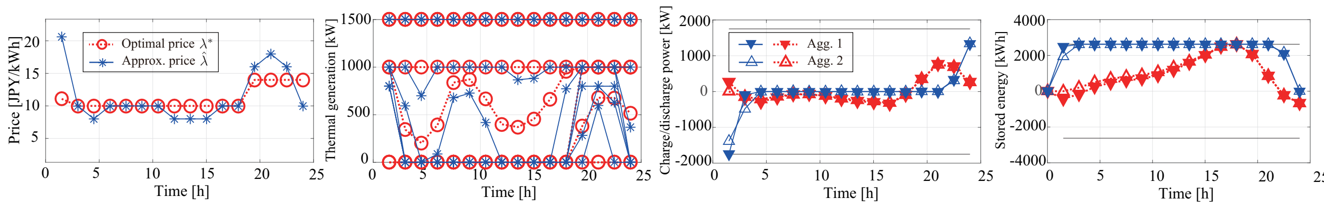

Fig. 2. Price profiles, dispatchable power generation profiles, charge and discharge power profiles, and stored energy profiles in 10% penetration level.

case are all separate, i.e., $\hat{\boldsymbol{x}}_\alpha$ is identical to $\boldsymbol{x}_\alpha$ as shown in Theorem 1.

However, we further see that the deadweight loss as well as the resultant social cost increase as the level of energy storage penetration increases. This implies that the approximation of $\boldsymbol{x}_\alpha$ by $\hat{\boldsymbol{x}}_\alpha$ is not good enough when the level of energy storage penetration is high. In fact, as shown in Fig. 2, the resultant profiles of clearing prices and decision variables (the blue solid lines) have relatively large discrepancy from the optimal ones (the red dotted lines) even in the case of the 10% penetration level.

V. Bidding System for Energy Shiftability

A. Aggregator Model in Multiresolved Basis

In the rest of this paper, for simplicity of discussion, we suppose that n is a power of 2, i.e., $n = 2^N$ for a natural number N. Under this supposition, every index $h \in \mathcal{H}$ with

$$\mathcal{H} := \{0, 1, \dots, 2^N - 1\}$$

can be represented in the binary form of

$$h = \textstyle\sum_{j \in \mathcal{J}} \sigma_h^{(j)} 2^j, \quad \mathcal{J} := \{0, 1, \dots, N-1\}$$

where each $\sigma_h^{(j)}$ takes a binary value of 0 or 1. As being compatible with this binary representation, we consider a vector of coordinates given as

$$u_h := p_h^{(0)} \otimes p_h^{(1)} \otimes \cdots \otimes p_h^{(N-1)}, \quad h \in \mathcal{H}$$

where each $p_h^{(j)}$ is the binary vector of

$$p_h^{(j)} := \frac{1}{\sqrt{2}} \left(1 - \sigma_h^{(j)}\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{\sqrt{2}} \sigma_h^{(j)} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

The system of these coordinates forms an orthonormal basis denoted by $\{u_h\}_{h \in \mathcal{H}}$, which we call a *multiresolved basis*.

Note that u_h with a small number of h corresponds to a basis of low temporal resolution. On the other hand, u_h with a large number of h corresponds to a basis of high temporal resolution. For example,

$$u_0 = \left(\tfrac{1}{\sqrt{2}}\right)^N \mathbf{1}_{2^N}, \quad u_1 = \left(\tfrac{1}{\sqrt{2}}\right)^N \begin{pmatrix} \mathbf{1}_{2^{N-1}} \\ -\mathbf{1}_{2^{N-1}} \end{pmatrix} \tag{24}$$

correspond to the zero (lowest) frequency coordinate and the second lowest frequency coordinate, respectively. The basis transformation can be regarded as a bijection between the time domain and a frequency-like domain.

On the premise of the multiresolved basis, we consider formulating a transformed multiperiod electricity market with regard to *energy shiftability*. To this end, we represent a prosumption profile as $x = \sum_{h \in \mathcal{H}} w_h u_h$ where w_h denotes the hth component in the multiresolved basis. This is rewritten as $x = Uw$ where $w := (w_h)_{h \in \mathcal{H}}$ denotes the stacked component vector and

$$U := [u_0, u_1, \dots, u_{2^N - 1}] \tag{25}$$

denotes the unitary matrix associated with the multiresolved basis. With this basis transformation, we can define an alternative cost function $H : \mathbb{R}^{2^N} \to \mathbb{R}$ defined as

$$H(w) := F(Uw). \tag{26}$$

This transformed cost function represents the prosumption cost in the multiresolved basis. For example, its partial derivative with respect to w_0 represents the change rate of costs for increasing the total prosumption amount on the day of interest, and its partial derivative with respect to w_1 represents the change rate of costs for shifting prosumption amounts from the latter half (afternoon) timeslots towards the former half (morning) timeslots. They correspond to the coordinates u_0 and u_1 in (24).

B. Derivation of Separate Multidimensional Bid Functions

Consider the basis transformation of the price profile as $\lambda = U\eta$, where η denotes the transformed price profile in the multiresolved basis. Note that each price η_h is associated with the corresponding coordinate u_h. Then, the transformed profit function, which is equivalent to J in (7), is defined as

$$K(w; \eta) := \eta^\mathsf{T} w - H(w). \tag{27}$$

On the basis of this profit function, we can define the multidimensional bid function

$$\boldsymbol{w}(\eta) := \left\{ w : K(w; \eta) \geq K(w'; \eta),\ \forall w' \in U^\mathsf{T} \mathcal{X} \right\}, \tag{28}$$

which is given in the same manner as $\boldsymbol{x}$ in (8).

Let us consider deriving a separate approximant of the multidimensional bid function $\boldsymbol{w}$ according to the sequential procedure in Section IV-A. In the same manner as that for $\hat{\boldsymbol{x}}_t$ in (21), we can derive a bid function $\hat{\boldsymbol{w}}_h : \mathbb{R} \to \mathbb{R}$ associated with the hth resolution market. More specifically, under the supposition that a price prediction interval $[\hat{\eta}_{h+1:2^N-1}]$ is given and constants $\hat{w}_{0:h-1}$ and $\hat{\eta}_{0:h-1}$ are determined, we define a convex function $g_h : \mathbb{R} \to \mathbb{R}$ in the same manner as f_t in (17). Then, for the hth resolution profit function

$$\hat{K}_h(w_h; \eta_h) := \eta_h w_h - g_h(w_h), \tag{29}$$

we obtain the bid function as

$$\hat{\boldsymbol{w}}_h(\eta_h) := \left\{ w_h : \hat{K}_h(w_h; \eta_h) \geq \hat{K}_h(w_h'; \eta_h),\ \forall w_h' \in [\hat{w}_h] \right\}, \tag{30}$$

whose derivation is similar to that of (21).

Finally, we obtain a separate approximant $\hat{\boldsymbol{w}}:\mathbb{R}^{2^N}\to\mathbb{R}^{2^N}$ as the ensemble of $\hat{\boldsymbol{w}}_h$. Note that the original multidimensional bid functions $\boldsymbol{x}$ and $\boldsymbol{w}$ are equivalent up to the basis transformation, while the separate approximants $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{w}}$ are not equivalent. Thus, the resultant values of deadweight loss are different in general.

C. Energy Shiftability Market Clearing

Consider a set of aggregators. As shown in [3], the optimal prosumption profile x_α^* in (12) of at least one aggregator is *shiftable*, i.e., the directional derivative [8] of F_α at x_α^* with respect to e_i-e_j is zero for every pair $(i,j)\in\mathcal{T}\times\mathcal{T}$, if and only if the optimal price profile λ^* in (12) lies in the image of u_0 in (24), i.e., $\lambda_1^*=\cdots=\lambda_n^*$. This implies that the optimal clearing price profile levels off due to the shiftability of prosumption profiles. Note that this price leveling off is expected when the level of energy storage penetration is sufficiently high, because energy storage can be seen as a resource having a larger shiftable domain of prosumption profiles. The following theorem shows that, under such a situation of price leveling off, the transformed bidding system in Section V-B can realize the optimal clearing of the multiperiod electricity market.

Theorem 2: Consider a set of aggregators in Section II-A. For a price prediction interval such that $0\in[\hat{\eta}_{1:2^N-1}]$ holds, let $\hat{\boldsymbol{w}}_\alpha:\mathbb{R}^{2^N}\to\mathbb{R}^{2^N}$ denote the separate multidimensional bid function of the αth aggregator given as in (30). Then, with regard to the clearing price profile $\hat{\eta}$ such that

$$\hat{x}_{\mathcal{A}}=(U\hat{w}_\alpha)_{\alpha\in\mathcal{A}},\quad \hat{w}_{\mathcal{A}}\in\textstyle\prod_{\alpha\in\mathcal{A}}\hat{\boldsymbol{w}}_\alpha(\hat{\eta}) \tag{31}$$

is balancing, the optimal price profile λ^* in (12) is given by $\lambda^*=\hat{\eta}_0u_0$ if and only if $\hat{\eta}$ satisfies

$$\hat{\eta}_1=\hat{\eta}_2=\cdots=\hat{\eta}_{2^N-1}=0. \tag{32}$$

Furthermore, the deadweight loss with respect to $\hat{x}_{\mathcal{A}}$ is zero.

Proof: For simplicity of notation, we drop the subscript α as long as we can focus on one aggregator. We first prove that (32) is equivalent to

$$0\in\partial\hat{H}_h(\hat{w}_h),\quad \forall h\in\{h_1,h_2,\ldots,\bar{h}\} \tag{33}$$

where $\bar{h}:=2^N-1$ and $\hat{H}_h(w_h):=H(\hat{w}_{0:h-1},w_h,\hat{w}_{h+1:\bar{h}})$, provided that $\hat{w}\in\hat{\boldsymbol{w}}(\hat{\eta})$ holds as in (31). This proof is done by induction in the descending order. Initially, we have

$$\hat{K}_{\bar{h}}(w_{\bar{h}};\eta_{\bar{h}})=w_{\bar{h}}\eta_{\bar{h}}-H(\hat{w}_{0:\bar{h}-1},w_{\bar{h}})$$

for $\hat{K}_h$ in (29). Owing to the convexity of $H(\hat{w}_{0:\bar{h}-1},\cdot)$, the condition of $\hat{\eta}_{\bar{h}}=0$ is equivalent to the fact that the $\bar{h}$th element $\hat{w}_{\bar{h}}$ maximizes $\hat{K}_{\bar{h}}(\cdot;0)$, namely $0\in\partial\hat{H}_h(\hat{w}_h)$ for $h=\bar{h}$. Next, supposing that

$$0\in\partial\hat{H}_h(\hat{w}_h),\quad \forall h\in\{\bar{h}-\theta+1,\ldots,\bar{h}\} \tag{34}$$

for $\theta=i$, we show that (34) holds again for $\theta=i+1$. Note that $\hat{\eta}_{\bar{h}-i}=0$ is equivalent to the fact that $\hat{w}_{\bar{h}-i}$ satisfies $0\in\partial g_{\bar{h}-i}(\hat{w}_{\bar{h}-i})$ where

$$\begin{aligned} g_{\bar{h}-i}(w_{\bar{h}-i})=-\min_{\eta_{\bar{h}-i+1:\bar{h}}}\max_{w_{\bar{h}-i+1:\bar{h}}}\big\{&\eta_{\bar{h}-i+1:\bar{h}}^{\mathsf{T}}w_{\bar{h}-i+1:\bar{h}}\\ &-H(\hat{w}_{0:\bar{h}-i-1},w_{\bar{h}-i},w_{\bar{h}-i+1:\bar{h}})\big\}. \end{aligned} \tag{35}$$

Furthermore, provided that the price prediction interval $[\hat{\eta}_{\bar{h}-i+1:\bar{h}}]$, which is the domain of the minimization with respect to $\eta_{\bar{h}-i+1:\bar{h}}$ in (35), involves 0 as a feasible element, the convex program inside $g_{\bar{h}-i}(\hat{w}_{\bar{h}-i})$ has the solution of

$$\eta_{\bar{h}-i+1:\bar{h}}=0,\quad w_{\bar{h}-i+1:\bar{h}}=\hat{w}_{\bar{h}-i+1:\bar{h}}.$$

This is ensured by (34) with $\theta=i$. Therefore, we see that

$$0\in\partial g_{\bar{h}-i}(\hat{w}_{\bar{h}-i})=\partial\hat{H}_h(\hat{w}_h),$$

which proves (34) for $\theta=i+1$. Hence, the equivalence between (32) and (33) is proven.

Notice that the optimal multidimensional bid function $\boldsymbol{w}$ in (28) is given as $\boldsymbol{w}=\partial\overline{H}$, where $\overline{H}$ denotes the conjugate of H. For the arguments below, let us confine our attention on the domain of $\eta\in\operatorname{im}e_0$, where we denote the unit vector associated with η_0 by e_0. Then, for any $\hat{w}_0\in e_0^{\mathsf{T}}\partial\overline{H}(e_0\eta_0)$, it follows that

$$\boldsymbol{w}(e_0\eta_0)=\begin{pmatrix} e_0^{\mathsf{T}}\partial\overline{H}(e_0\eta_0)\\ \partial\overline{H}(\hat{w}_0,0)\end{pmatrix}$$

where $\overline{H}(\hat{w}_0,\cdot)$ denotes the conjugate of $H(\hat{w}_0,\cdot)$. In the following, we show that

$$\boldsymbol{w}(e_0\eta_0)=\big(\hat{\boldsymbol{w}}_0(\eta_0),\hat{\boldsymbol{w}}_1(0),\ldots,\hat{\boldsymbol{w}}_{\bar{h}}(0)\big)^{\mathsf{T}}. \tag{36}$$

Note that the minimization in (35) has the solution being equal to zero if (32) or equivalently (33) holds. Thus, without loss of generality, we can assume that $[\hat{\eta}_{1:\bar{h}}]=0$ for its feasible domain. This reduces the formula of $\hat{K}_h$ in (29) as

$$\hat{K}_h(w_h;\eta_h)=w_h\eta_h-\max_{w_{h+1:\bar{h}}}\{-H(\hat{w}_{0:h-1},w_{h:\bar{h}})\}. \tag{37}$$

Let us first consider the case of $h=0$. From the reduced formula, we see that $\hat{\boldsymbol{w}}_0(e_0\eta_0)$, which is given as the set of w_0 attaining the maximum of $\hat{K}_0(\cdot;e_0\eta_0)$, can be expressed as $\hat{\boldsymbol{w}}_0(\eta_0)=e_0^{\mathsf{T}}\partial\overline{H}(e_0\eta_0)$, which proves the identity of the zeroth element of (36).

On the other hand, because the maximization in (37) has a solution of $\hat{w}_{h+1:\bar{h}}$, $\hat{\boldsymbol{w}}_h(0)$ for each $h\geq 1$ is given as the set of w_h such that $0\in\partial H_h(w_h)$. This is satisfied for $\hat{w}_h$ because of (33). Therefore, stacking them for all $h\geq 1$, we have $(\hat{\boldsymbol{w}}_1(0),\ldots,\hat{\boldsymbol{w}}_{\bar{h}}(0))^{\mathsf{T}}=\partial\overline{H}(\hat{w}_0,0)$, which proves the identity of the remaining elements of (36).

Let us denote the left-hand side of (36) associated with the αth aggregator by $\boldsymbol{w}_\alpha(e_0\eta_0)$. In this notation, the balance equation in (31) with (32) implies that

$$\exists w_{\mathcal{A}}^*\in\textstyle\prod_{\alpha\in\mathcal{A}}\boldsymbol{w}_\alpha(e_0\hat{\eta}_0)\ \text{ s.t. }\ \sum_{\alpha\in\mathcal{A}}w_\alpha^*=0. \tag{38}$$

Therefore, from the uniqueness of the optimal price profile, we see that η^* satisfying $0\in\sum_{\alpha\in\mathcal{A}}\boldsymbol{w}_\alpha(\eta^*)$ is given by $\eta^*=e_0\hat{\eta}_0$. Hence, $\lambda^*=\hat{\eta}_0u_0$ is proven. The deadweight loss of $\hat{x}_{\mathcal{A}}$ is zero because (38) is equivalent to (12). ■

Theorem 2 implies that, in a situation where the optimal price profile λ^* levels off, the optimal price profile is obtained as $\lambda_1^*=\cdots=\lambda_n^*=(\frac{1}{\sqrt{2}})^N\hat{\eta}_0$, where $\hat{\eta}_0$ denotes the clearing price for the total prosumption on the day. As discussed in [3], such price levelling off can be expected in high penetration of energy storage. From this viewpoint, the

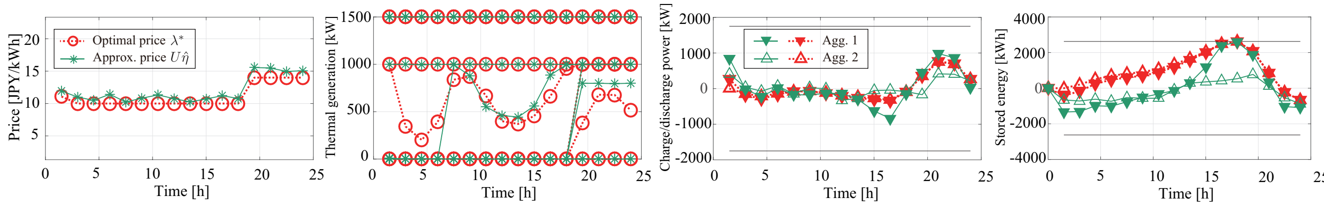

Fig. 3. Price profiles, dispatchable power generation profiles, charge and discharge power profiles, and stored energy profiles in 10% penetration level.

sequential determination of separate multiperiod bid functions in the multiresolved basis can work well for clearing the multiperiod electricity market under high energy storage penetration. Note that the sequential determination does not work well in the original time basis as demonstrated in Section IV-C.

D. Numerical Example

Consider the same setting as that in Section IV-C. We give the price prediction interval $[\hat{\eta}_{1:15}]$ as the multidimensional interval from -9.5 to 9.1. Varying the levels of energy storage penetration, we calculate the resultant deadweight loss for the transformed bidding system in Section V-B. The resultant social cost $\sum_{\alpha \in \mathcal{A}} H_\alpha(\hat{w}_\alpha)$ is over plotted in Fig. 1(b) by the green solid line with diamonds. From this figure, we see that the resultant social cost decreases as the penetration level increases. Furthermore, the deadweight loss, represented as the difference from the red dotted line with circles, is smaller than the deadweight loss in Section IV-C. In fact, in the case of no energy storage penetration, i.e., the 0% penetration level, the resultant deadweight loss is shown to be zero because the separate approximant $\hat{\boldsymbol{w}}_\alpha$ is identical to the original multidimensional bid function $\boldsymbol{w}_\alpha$ for every $\alpha \in \{1, 2, 3\}$. In addition, as expected from Theorem 2, the deadweight loss becomes smaller as the penetration level becomes higher. For reference, the resultant profiles of clearing prices and decision variables in the case of the 10% penetration level are plotted in Fig. 3, where we use the same legends as those in Fig 2. From these results, we see that the distributed approximate scheme for market clearing has good compatibility with the bidding system in the multiresolved basis.

VI. Concluding Remarks

In this paper, we have designed a bidding system for a multiperiod electricity market with consideration of the pricing of the shiftability of energy storage. More specifically, we have first developed a distributed approximate scheme for multiperiod electricity market clearing. Then, based on a basis transformation similar to the Fourier transformation, we propose a bidding system with explicit consideration of the pricing of energy shiftability. It has been theoretically and numerically shown that the distributed approximate scheme has good compatibility with the bidding system in the Fourier-like basis. Generalization to the integration of renewable power generation, which may have a steep fluctuation, is a meaningful future work to pursue.

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Appendix

We overview several facts from convex analysis theory [8]. The *conjugate* of a convex function F is defined by

$$\overline{F}(\lambda) := \sup_{x \in \mathrm{dom}\, F} \left\{ \lambda^{\mathsf{T}} x - F(x) \right\}, \tag{39}$$

where $\mathrm{dom}\, F$ denotes the effective domain of F. It is known that $\overline{F}$ is convex and the conjugate of $\overline{F}$ coincides with F as long as F is convex. Furthermore, the strict convexity of F is equivalent to the smoothness of $\overline{F}$. The transformation in (39) is called the Legendre-Fenchel transformation.

The subdifferential of a convex function F is defined by

$$\partial F(x) := \{\lambda : F(x') \geq F(x) + \lambda^{\mathsf{T}}(x' - x),\ \forall x' \in \mathrm{dom}\, F\},$$

which is a set-valued function with a convex image. Corollary 31.5.2 of [8] shows that $\partial F : \mathbb{R}^n \to \mathbb{R}^n$ is monotone increasing if F is convex. In particular, it is shown to be strictly monotone increasing if F is strictly convex; see Theorem 2.1 in [9]. Furthermore, $x \in \partial \overline{F}(\lambda)$ and $\lambda \in \partial F(x)$ are equivalent as shown in Theorem 23.5 in [8].