Workshop COOPS 2017 in Milano

Bidding System Design for Multiperiod Electricity Markets

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Contents

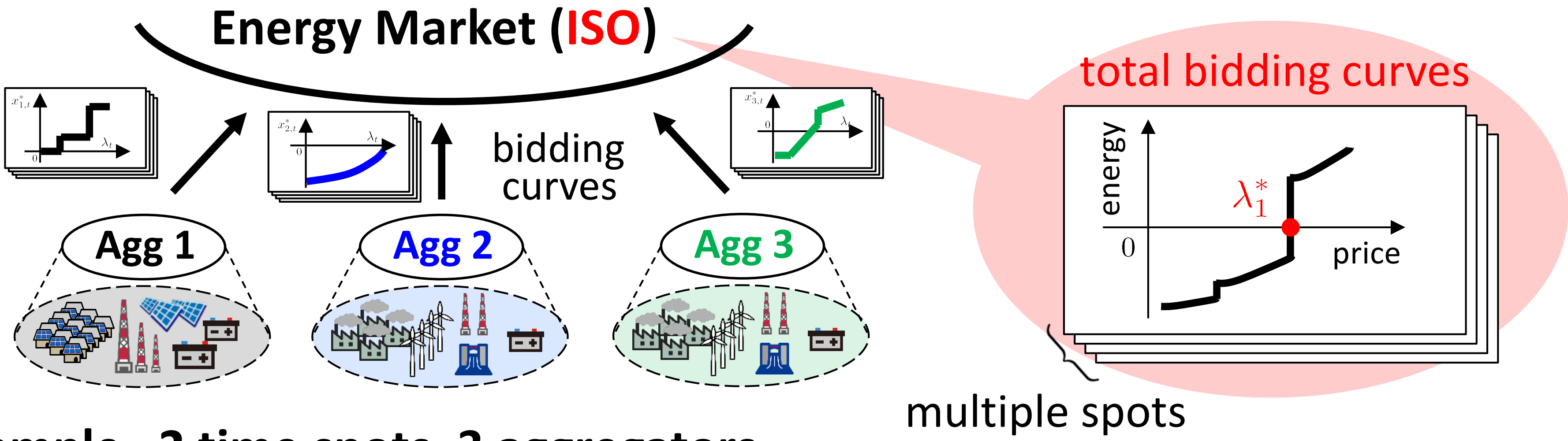
Problem formulation

- **Part I: What is bidding system design?**
  - relation between **market clearing problem** and **convex optimization**
  - relation between **bidding curves** and **cost functions**
  - difficulty in bidding system design for **multiperiod markets**

- **Part II: How to design a bidding system for multiperiod markets?**
  - basis transformation compatible with **energy shift market**
  - **sequential market clearing scheme**
  - numerical examples

An approximate solution method

Day-Ahead Energy Markets



Example 2 time spots, 3 aggregators

Market Results	Aggregator 1 (producer)	Aggregator 2 (consumer)	Aggregator 3 (prosumer)	Clearing Price
Spot 1 (AM)	150 [kWh]	-250 [kWh]	100 [kWh]	10 [yen/kWh]
Spot 2 (PM)	100 [kWh]	-50 [kWh]	-50 [kWh]	5 [yen/kWh]

Decision variables:

Balanced

$$\boxed{x_1^* = \begin{pmatrix} 150 \\ 100 \end{pmatrix} \quad x_2^* = \begin{pmatrix} -250 \\ -50 \end{pmatrix} \quad x_3^* = \begin{pmatrix} 100 \\ -50 \end{pmatrix}} \qquad \lambda^* = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

Market clearing: Find "desirable" λ^* & $(x_\alpha^*)_{\alpha \in \mathcal{A}}$ such that $\sum_{\alpha \in \mathcal{A}} x_\alpha^* = 0$

Market Clearing as Optimization

Market Results	Aggregator 1 (producer)	Aggregator 2 (consumer)	Aggregator 3 (prosumer)	Clearing Price
Spot 1 (AM)	150 [kWh]	-250 [kWh]	100 [kWh]	10 [yen/kWh]
Spot 2 (PM)	100 [kWh]	-50 [kWh]	-50 [kWh]	5 [yen/kWh]

x_1^* λ^*

Profit of Agg α: $J_\alpha(x_\alpha^*;\lambda^*) = \underbrace{\langle \lambda^*, x_\alpha^* \rangle}_{\text{income}} - \underbrace{F_\alpha(x_\alpha^*)}_{\text{cost}}$

(selfish objective function)

✓ See later how to determine $F_\alpha(x_\alpha)$

(especially for prosumer!)

Social profit: $\sum_{\alpha\in\mathcal{A}} J_\alpha(x_\alpha^*;\lambda^*) = \left\langle \lambda^*, \underbrace{\cancel{\sum_{\alpha\in\mathcal{A}} x_\alpha^*}}_{=0} \right\rangle - \underbrace{\sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha^*)}_{\text{social cost}}$

(social objective function)

Social profit maximization = Social cost minimization

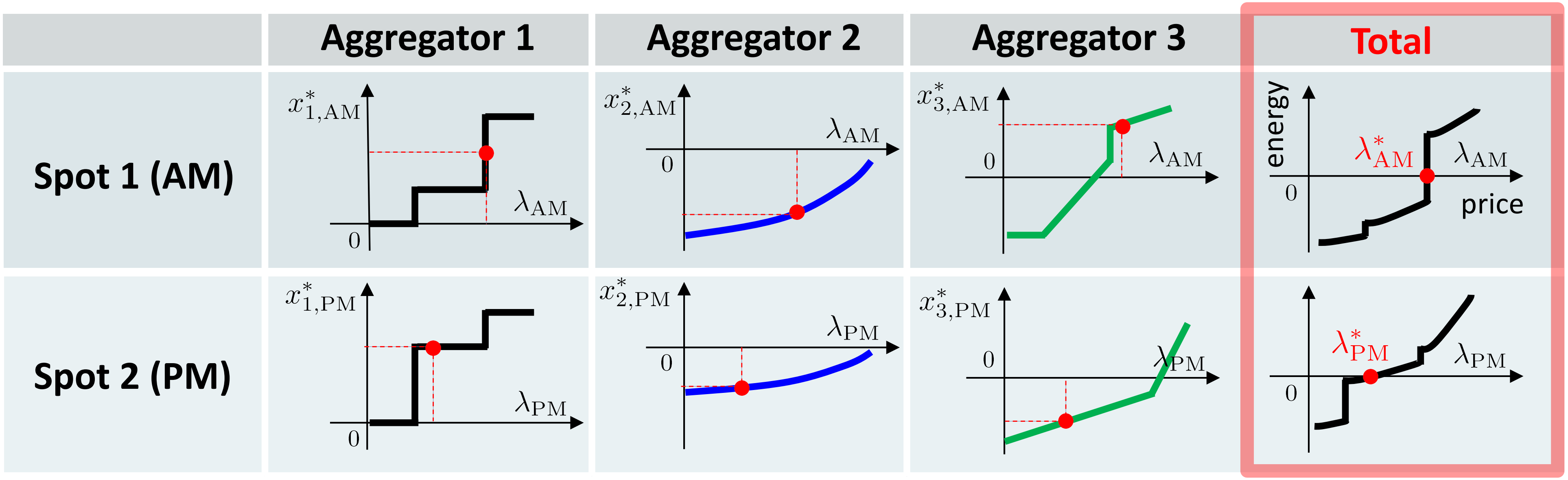
$$\min_{(x_\alpha)_{\alpha\in\mathcal{A}}} \sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha) \quad \text{s.t.} \quad \sum_{\alpha\in\mathcal{A}} x_\alpha = 0$$

primal: $(x_\alpha^*)_{\alpha\in\mathcal{A}}$

dual: λ^*

Bidding Curves for Market Clearing

Suppose that bidding curves for each spot are submitted to ISO



ISO can find each **clearing price** and **balancing amounts** as **crossing points of (total) bidding curves**

But… Are such crossing points really solutions of social cost minimization:

$$\min_{(x_\alpha)_{\alpha\in\mathcal{A}}} \sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha) \quad \text{s.t.} \quad \sum_{\alpha\in\mathcal{A}} x_\alpha = 0 \quad ??$$

Brief Summary: Bidding System Design

Socially optimal market clearing problem:

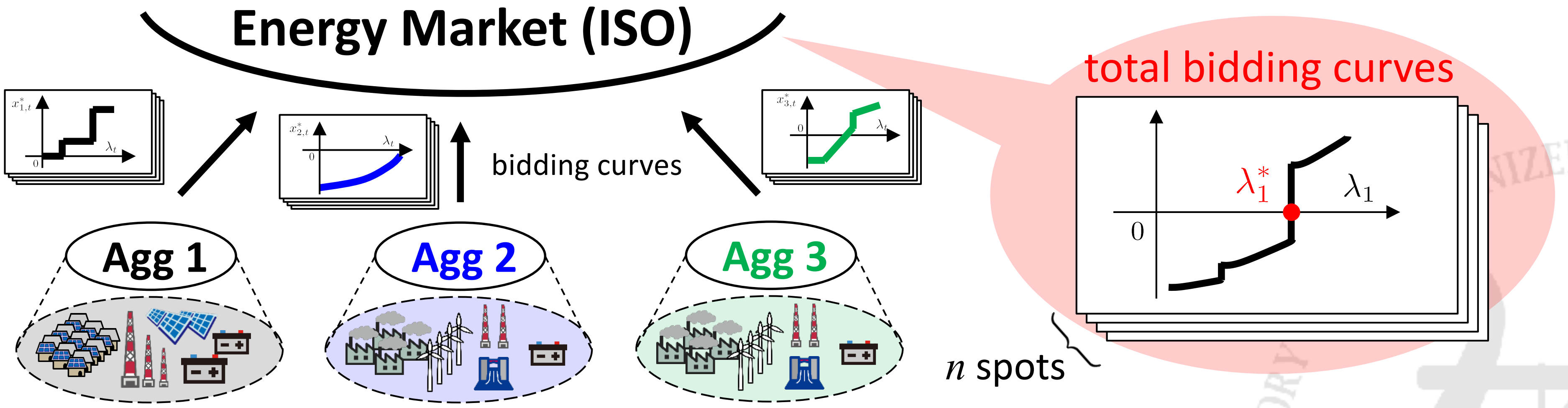
Find $\begin{cases} \text{primal: } (x_\alpha^*)_{\alpha\in\mathcal{A}} \\ \text{dual: } \lambda^* \end{cases}$ solving $\min_{(x_\alpha)_{\alpha\in\mathcal{A}}} \sum_{\alpha\in\mathcal{A}} F_\alpha(x_\alpha)$ s.t. $\sum_{\alpha\in\mathcal{A}} x_\alpha = 0$

Q1: What is a reasonable cost function $F_\alpha(x_\alpha)$??

✓ Prosumption x_α should be a **mixture of generators, batteries, renewables** etc

Q2: Is it possible to construct bidding curves from $F_\alpha(x_\alpha)$??

✓ **Social cost should be minimized** with $(x_\alpha^*)_{\alpha\in\mathcal{A}}$ and λ^* found as crossing-points



✓ **Bidding system design = Distributed algorithm design under pre-specified ISO operation**

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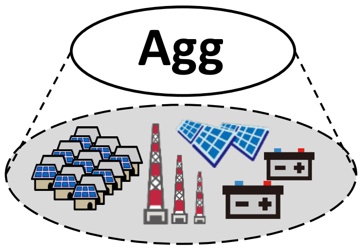
Prosumption Cost function



$$x = g - l + \eta^{\text{out}}\delta^{\text{out}} - \frac{1}{\eta^{\text{in}}}\delta^{\text{in}}$$

Internal decision variables

- generated power g
- battery charge/discharge $\delta^{\text{in}}, \delta^{\text{out}}$

✓ **Constraints**: $g \in \mathcal{G},\ \delta \in \mathcal{D}$
(e.g. $0 \le g \le \overline{g}$)

Given

Generation cost	$G(g)$
Battery usage cost	$D(\delta)$

【Theorem】 If $G(g)$ and $D(\delta)$ are both convex, then

$$F(x) = \min_{(g,\delta)\in\mathcal{F}(x)} \left\{ G(g) + D(\delta) \right\}$$ is convex with respect to x

where $\mathcal{F}(x) := \left\{ (g,\delta) \in \mathcal{G} \times \mathcal{D} : x = g - l + \eta^{\text{out}}\delta^{\text{out}} - \frac{1}{\eta^{\text{in}}}\delta^{\text{in}} \right\}$

Example **2 time spots** (AM/PM) **Constants:** $l_{\text{AM}} = 50,\ l_{\text{PM}} = 10,\ \eta^{\text{out}} = \eta^{\text{in}} = 1$

$$F(x_{\text{AM}}, x_{\text{PM}}) = \min_{g,\delta} \left\{ G(g) + D(\delta) \right\} \quad \text{s.t.} \quad \begin{pmatrix} x_{\text{AM}} \\ x_{\text{PM}} \end{pmatrix} = \begin{pmatrix} g_{\text{AM}} - 50 + \delta^{\text{out}}_{\text{AM}} - \delta^{\text{in}}_{\text{AM}} \\ g_{\text{PM}} - 10 + \delta^{\text{out}}_{\text{PM}} - \delta^{\text{in}}_{\text{PM}} \end{pmatrix}$$

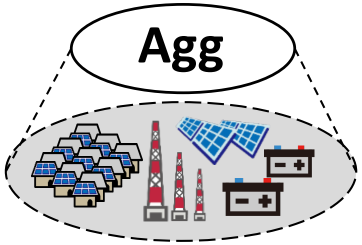
g, δ not unique!

✓ $x = 0$: supply-demand balance inside aggregator

Optimal strategy for energy resources ensures convexity of $F(x)$

Prosumption Cost function

$$x = g - l + \eta^{\text{out}}\delta^{\text{out}} - \frac{1}{\eta^{\text{in}}}\delta^{\text{in}}$$



Internal decision variables

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✓ **Uncertain renewables** can be handled as robust optimization like:

$$F(x) = \max_{p\in\mathcal{P}} \min_{(g,\delta)\in\mathcal{F}(x,p)} \left\{ G(g) + D(\delta) \right\}$$

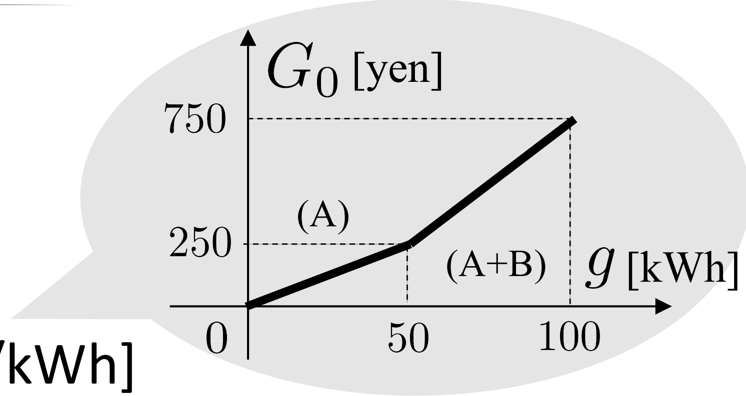
where $\mathcal{P}$ is a scenario set of renewable generation

on-going work

(More interesting to see **how magnitude of uncertainty affects economics!**)

Derivation of Bid Functions

Example **2 time spots** $l_{\rm AM} = 50,\ l_{\rm PM} = 50$



Generators & loads: $\begin{pmatrix} x_{\rm AM} \\ x_{\rm PM} \end{pmatrix} = \begin{pmatrix} g_{\rm AM} - l_{\rm AM} \\ g_{\rm PM} - l_{\rm PM} \end{pmatrix}$

Spec of generators: $\begin{cases} \text{(A) } 0\text{~}50 \text{ [kWh] } \ \ 5 \text{ [yen/kWh]} \\ \text{(B) } 0\text{~}50 \text{ [kWh] } \ 10 \text{ [yen/kWh]} \end{cases}$

$$\begin{cases} \text{Generation cost: } G(g_{\rm AM}, g_{\rm PM}) = G_0(g_{\rm AM}) + G_0(g_{\rm PM}) \\ \text{Feasible generator outputs: } 0 \le g_{\rm AM} \le 100 \quad 0 \le g_{\rm PM} \le 100 \end{cases}$$

additively decomposable

disjoint

$$\max_{x \in \mathcal{X}} J(x;\lambda) = \max_{x_{\rm AM} \in [-50,50]} \{\lambda_{\rm AM} x_{\rm AM} - G_0(x_{\rm AM} + 50)\} + \max_{x_{\rm PM} \in [-50,50]} \{\lambda_{\rm PM} x_{\rm PM} - G_0(x_{\rm PM} + 50)\}$$

decomposable!

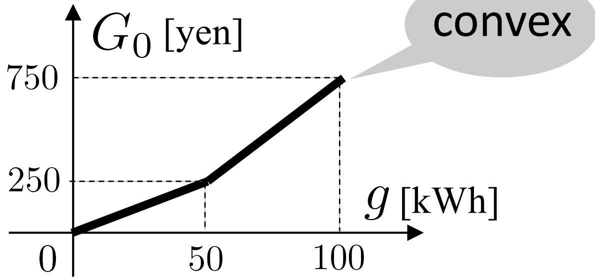
Bid functions

$$\boldsymbol{x}_t^*(\lambda_t) = \arg \max_{x_t \in [-50,50]} \{\lambda_t x_t - G_0(x_t + 50)\}, \quad t \in \{\rm AM, PM\}$$

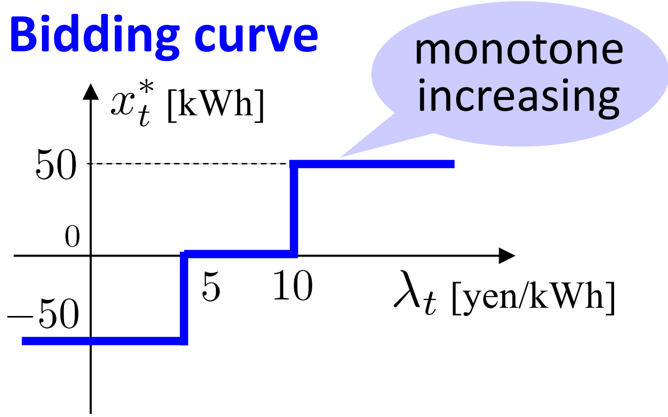
Mathematics behind Bid Functions

Bid functions (period-specific)

$$\boldsymbol{x}_t^*(\lambda_t) = \arg \max_{x_t \in [-50,50]} \{\lambda_t x_t - G_0(x_t + 50)\}$$

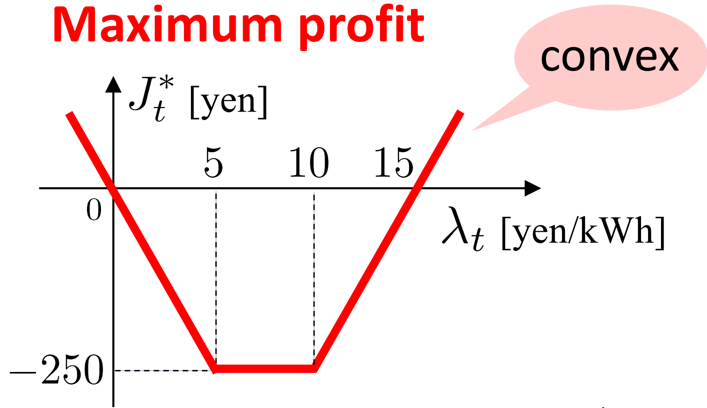


Bidding curve



$$\boldsymbol{x}_t^*(\lambda_t) = \partial J_t^*(\lambda_t)$$

Maximum profit



$$J_t^*(\lambda_t) = \max_{x_t \in [-50,50]} \{\lambda_t x_t - G_0(x_t + 50)\}$$

Legendre transform of cost function

【Legendre transformation】 (convex conjugation) ✓ $\overline{\overline{F}} = F \iff F$: convex

$$\overline{F}(\lambda) := \sup_{x \in \mathcal{X}} \{\langle \lambda, x \rangle - F(x)\}$$

Multiperiod Bid Function

Generation cost: $G(g_{\mathrm{AM}}, g_{\mathrm{PM}}) = G_0(g_{\mathrm{AM}}) + G_0(g_{\mathrm{PM}})$

Feasible generator outputs: $0 \le g_{\mathrm{AM}} \le 100 \quad 0 \le g_{\mathrm{PM}} \le 100$

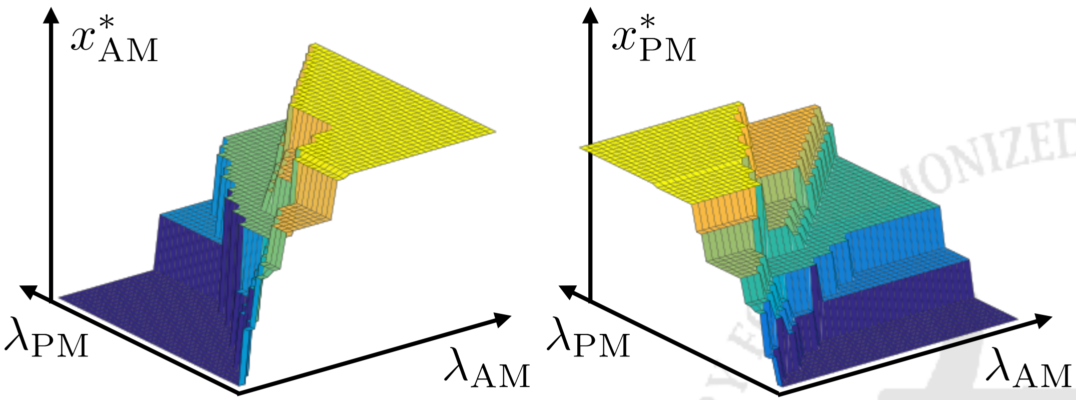
Ramp rate limit (Added): $-10 \le g_{\mathrm{AM}} - g_{\mathrm{PM}} \le 10$ **temporally correlated!**

Multiperiod bid function

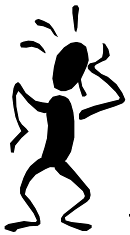
indecomposable!

$$\boldsymbol{x}^*(\lambda) = \arg\max_{x \in \mathcal{X}} \{\langle \lambda, x \rangle - F(x)\} = \begin{pmatrix} \boldsymbol{x}^*_{\mathrm{AM}}(\lambda_{\mathrm{AM}}, \lambda_{\mathrm{PM}}) \\ \boldsymbol{x}^*_{\mathrm{PM}}(\lambda_{\mathrm{AM}}, \lambda_{\mathrm{PM}}) \end{pmatrix}$$

✓ $\boldsymbol{x}^* = \partial \overline{F}$: monotone increasing



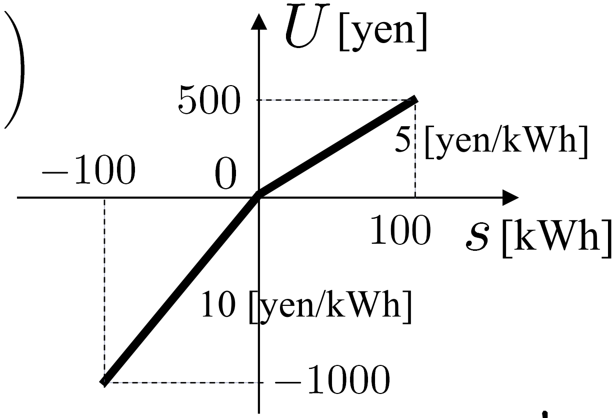
~~bidding curves~~ ➞ **bidding hyperplanes**



Multiperiod bid function is **not compatible with current bidding system**

Separability of Multiperiod Bid Function

<u>Example</u> Battery aggregator $\begin{pmatrix} x_{\mathrm{AM}} \\ x_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$



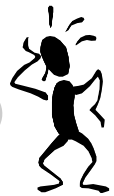
SOC constraints: $\delta \in \mathcal{D}_{\mathrm{SOC}}$

not disjoint!

Cost function based on utility of final SOC:

$$D(\delta) = -U\big(s_{\mathrm{fin}}(\delta)\big) \quad s_{\mathrm{fin}}(\delta) = s_0 + \sum_{t \in \{\mathrm{AM},\mathrm{PM}\}} (\delta^{\mathrm{in}}_t - \delta^{\mathrm{out}}_t)$$

not additively decomposable!



【Lemma】 The multiperiod bid function is *separate* iff

the cost function is *additively decomposable* and its domain is *disjoint*

i.e. $$\boldsymbol{x}^*(\lambda) = \begin{pmatrix} \boldsymbol{x}^*_1(\lambda_1) \\ \vdots \\ \boldsymbol{x}^*_n(\lambda_n) \end{pmatrix} \iff F(x) = \sum_{t=1}^{n} F_t(x_t), \quad x \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_n.$$

<u>Negative fact!!</u> Traditional bidding curves available **just in very special cases**

Brief Summary: Bidding System Design

Socially optimal market clearing : $$\min_{(x_\alpha)_{\alpha \in \mathcal{A}}} \sum_{\alpha \in \mathcal{A}} F_\alpha(x_\alpha) \quad \text{s.t.} \quad \sum_{\alpha \in \mathcal{A}} x_\alpha = 0$$

✓ **Bidding system design = Distributed algorithm design under pre-specified ISO operation**

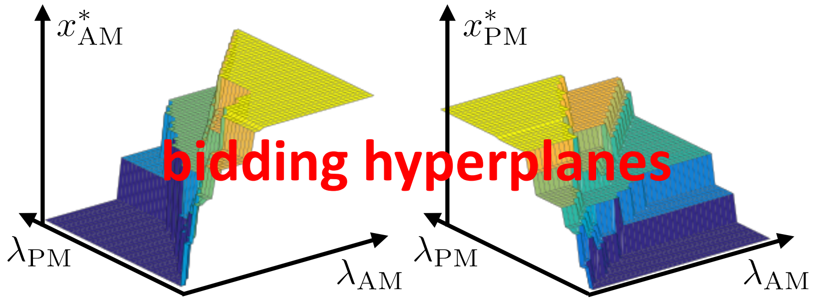
【Theorem】 $$F(x) = \min_{(g,\delta) \in \mathcal{F}(x)} \left\{ G(g) + D(\delta) \right\}$$ is convex

where $$\mathcal{F}(x) := \left\{ (g, \delta) \in \mathcal{G} \times \mathcal{D} : x = g - l + \eta^{\mathrm{out}} \delta^{\mathrm{out}} - \frac{1}{\eta^{\mathrm{in}}} \delta^{\mathrm{in}} \right\}$$

Multiperiod bid function:

$$\boldsymbol{x}^*(\lambda) = \arg\max_{x \in \mathcal{X}} \{ \langle \lambda, x \rangle - F(x) \}$$

✓ monotone increasing $\boldsymbol{x}^* = \partial \overline{F} : \mathbb{R}^n \to \mathbb{R}^n$



【Lemma】

additively decomposable

$$\boldsymbol{x}^*(\lambda) = \begin{pmatrix} \boldsymbol{x}_1^*(\lambda_1) \\ \vdots \\ \boldsymbol{x}_n^*(\lambda_n) \end{pmatrix} \iff F(x) = \sum_{t=1}^{n} F_t(x_t), \quad x \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_n$$

separate   disjoint

<u>Bidding system design for multiperiod markets is not so simple!!</u>

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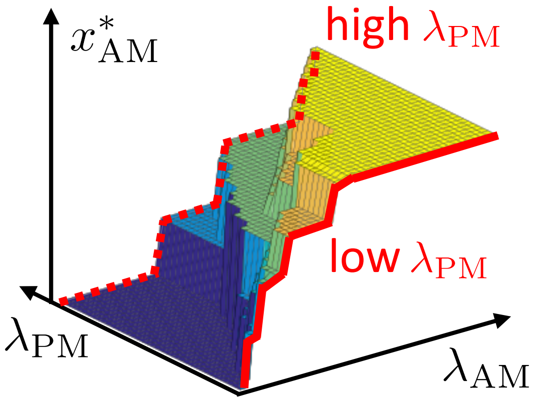
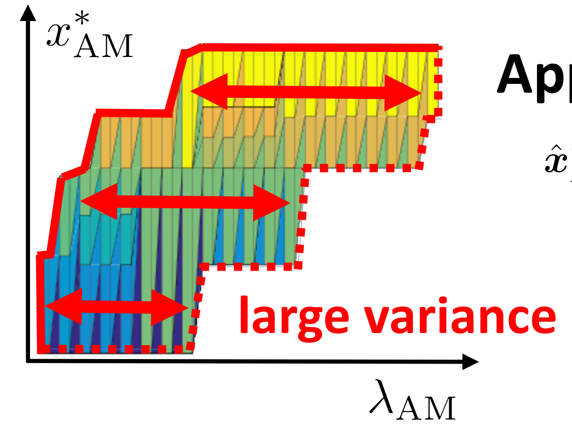
Ideas in Proposed Approach

A) Basis transformation towards better approximation

B) Approximation of bidding hyperplanes to bidding curves



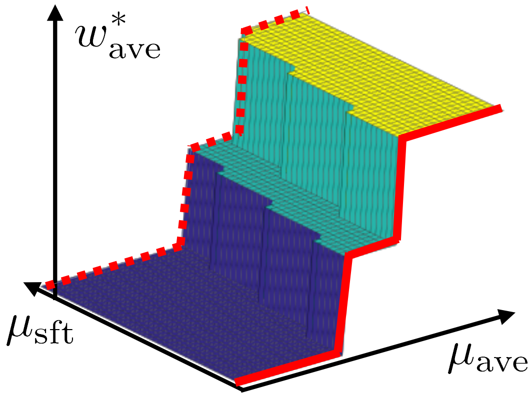


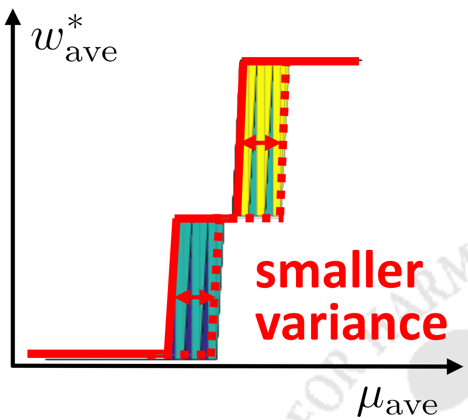
Approximate bidding curve

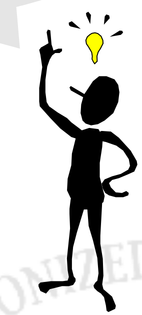
$$\hat{\boldsymbol{x}}^*_{\rm AM}(\hat{\lambda}_{\rm AM}) = \boldsymbol{x}^*_{\rm AM}(\hat{\lambda}_{\rm AM}, \lambda_{\rm PM})\Big|_{\lambda_{\rm PM}=\hat{\lambda}_{\rm PM}}$$

$$\begin{pmatrix} x_{\rm AM} \\ x_{\rm PM} \end{pmatrix} \overset{T}{\longmapsto} \begin{pmatrix} w_{\rm ave} \\ w_{\rm sft} \end{pmatrix}$$

$$\begin{pmatrix} \lambda_{\rm AM} \\ \lambda_{\rm PM} \end{pmatrix} \overset{T}{\longmapsto} \begin{pmatrix} \mu_{\rm ave} \\ \mu_{\rm sft} \end{pmatrix}$$







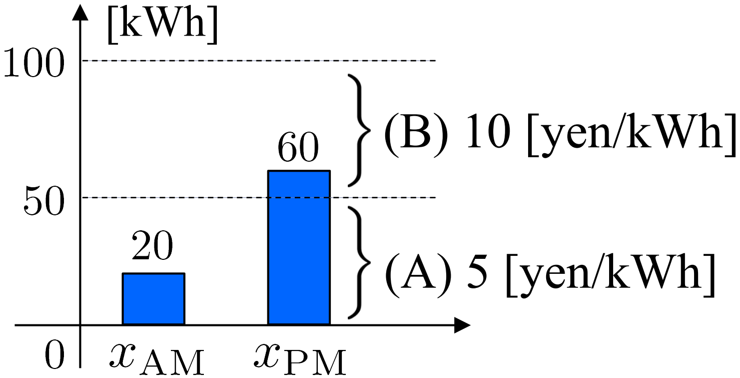
C) Sequential market clearing scheme

✓ From optimization view: (A) preconditioning (B)-(C) updates of primal/dual variables

Energy Shift: A Key Property of Batteries

Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$

Spec of gens: $\begin{cases} \text{(A)} \ \ 0\sim50 \ [\text{kWh}] \quad 5 \ [\text{yen/kWh}] \\ \text{(B)} \ \ 0\sim50 \ [\text{kWh}] \quad 10 \ [\text{yen/kWh}] \end{cases}$



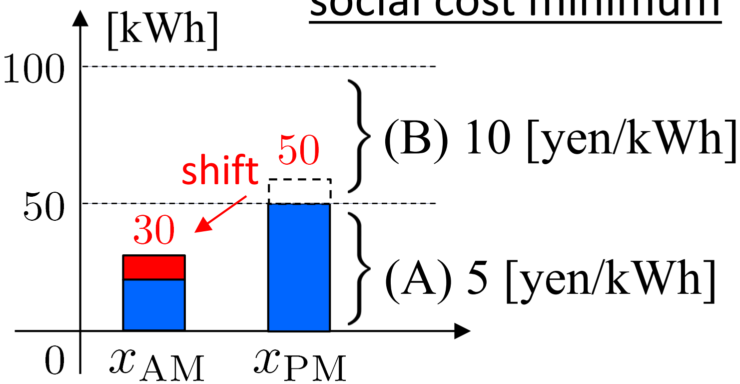
Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

Optimal price: $\begin{pmatrix} \lambda^*_{\mathrm{AM}} \\ \lambda^*_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

Agg 3 (batteries): $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$

$\lambda^*_{\mathrm{PM}} > \lambda^*_{\mathrm{AM}} \longrightarrow$
- PM: **discharge** (sell)
- AM: **charge** (buy)

<u>social cost minimum</u>



New optimal price: $\begin{pmatrix} \lambda^*_{\mathrm{AM}} \\ \lambda^*_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

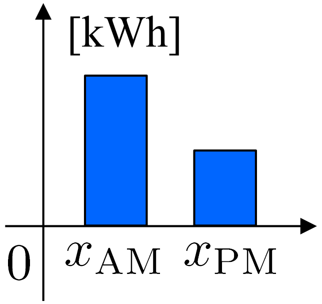
<u>**Battery leads to price levelling-off!**</u>

<u>**Energy market with explicit consideration of energy shift??**</u>

Fourier-Like Basis Transformation

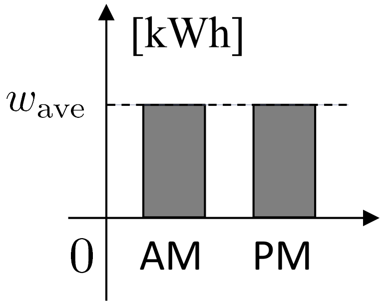
Example **2 time spots**

$$w_{\mathrm{ave}} = \frac{x_{\mathrm{AM}}+x_{\mathrm{PM}}}{2} \qquad w_{\mathrm{sft}} = \frac{x_{\mathrm{AM}}-x_{\mathrm{PM}}}{2}$$



$$\begin{pmatrix} x_{\mathrm{AM}} \\ x_{\mathrm{PM}} \end{pmatrix}$$

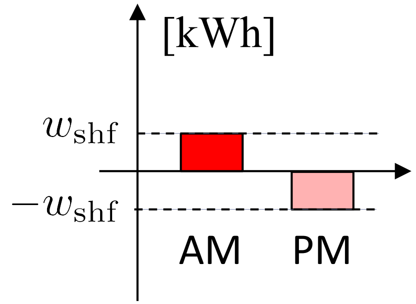
$=$



$$w_{\mathrm{ave}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

average (total) energy

$+$



$$w_{\mathrm{sft}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

shift energy (PM to AM)

$$\begin{pmatrix} \lambda_{\mathrm{AM}} \\ \lambda_{\mathrm{PM}} \end{pmatrix} = \mu_{\mathrm{ave}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \mu_{\mathrm{sft}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- μ_{ave} : **average (levelling-off) price**
- μ_{sft} : **energy shift price**

Explicit consideration of levelling-off price & energy shift price

Energy shift market: $\min_{(w_\alpha)_{\alpha\in\mathcal{A}}} \sum_{\alpha\in\mathcal{A}} H_\alpha(w_\alpha) \ \text{s.t.} \ \sum_{\alpha\in\mathcal{A}} w_\alpha = 0$

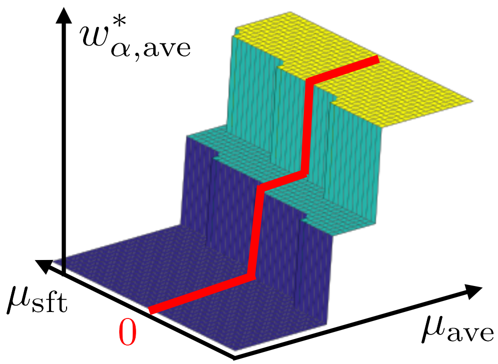
- **primal:** $(w_\alpha^*)_{\alpha\in\mathcal{A}}$
- **dual:** μ^*

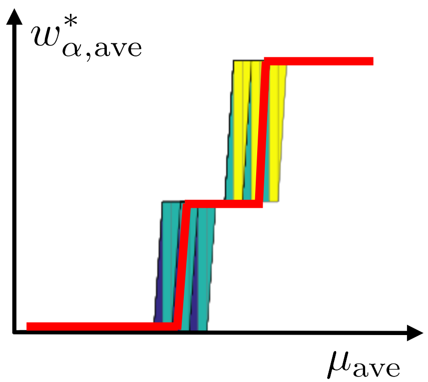
Sequential Market Clearing

Step 1) Market clearing of average energy amounts

Multiperiod bid function: $\begin{pmatrix} \boldsymbol{w}^*_{\alpha,\mathrm{ave}}(\mu_{\mathrm{ave}}, \mu_{\mathrm{sft}}) \\ \boldsymbol{w}^*_{\alpha,\mathrm{sft}}(\mu_{\mathrm{ave}}, \mu_{\mathrm{sft}}) \end{pmatrix} = \arg\max_{w_\alpha} \{ \langle \mu, w_\alpha \rangle - H_\alpha(w_\alpha) \}$

Approximate bid function: $\hat{w}^*_{\alpha,\mathrm{ave}}(\hat{\mu}_{\mathrm{ave}}) = \boldsymbol{w}^*_{\alpha,\mathrm{ave}}(\hat{\mu}_{\mathrm{ave}}, \mu_{\mathrm{sft}})\Big|_{\mu_{\mathrm{sft}}=0}$





assumption (premise) of
price levelling-off

$$\begin{pmatrix} \lambda_{\mathrm{AM}} \\ \lambda_{\mathrm{PM}} \end{pmatrix} = \mu_{\mathrm{ave}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \cancel{\mu_{\mathrm{sft}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

⟶ ISO determines $\hat{\mu}^*_{\mathrm{ave}}$ & $(\hat{w}^*_{\alpha,\mathrm{ave}})_{\alpha\in\mathcal{A}}$ by approximate bidding curves

Step 2) Market clearing of shift energy amounts

cleared amount

Approximate bid function: $\hat{w}^*_{\alpha,\mathrm{sft}}(\hat{\mu}_{\mathrm{sft}}) = \arg\max_{w_{\alpha,\mathrm{sft}}} \{ \hat{\mu}_{\mathrm{sft}} w_{\alpha,\mathrm{sft}} - H_\alpha(\hat{w}^*_{\alpha,\mathrm{ave}}, w_{\alpha,\mathrm{sft}}) \}$

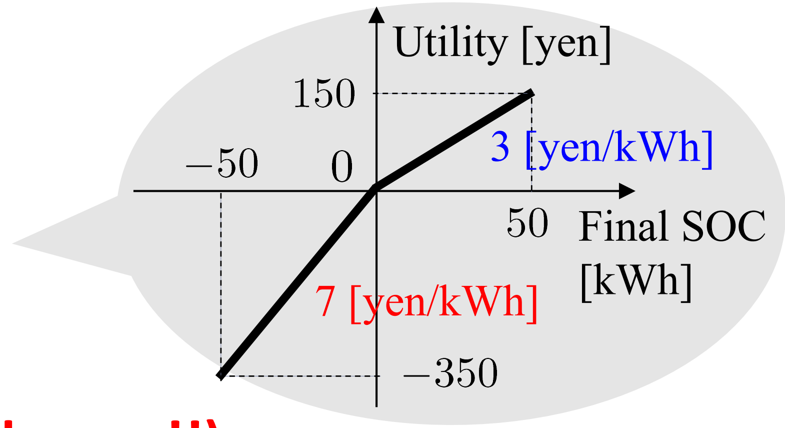
⟶ ISO determines $\hat{\mu}^*_{\mathrm{sft}}$ & $(\hat{w}^*_{\alpha,\mathrm{sft}})_{\alpha\in\mathcal{A}}$ by approximate bidding curves

Example: Sequential Market Clearing

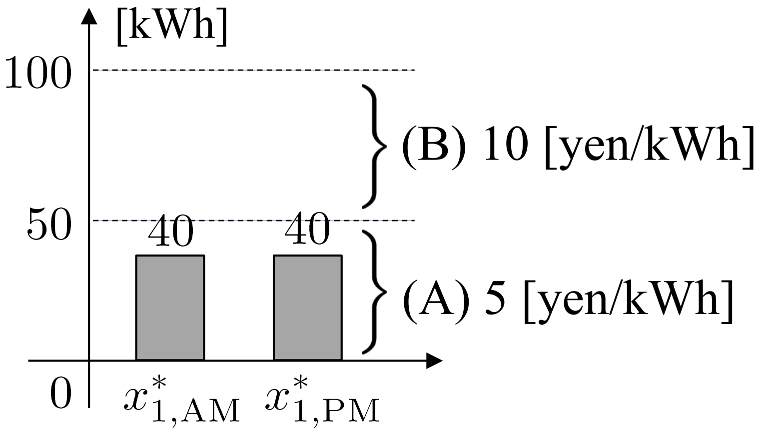
Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$ $\begin{cases} \text{(A)}\ \ 0\sim 50\ [\text{kWh}] \quad 5\ [\text{yen/kWh}] \\ \text{(B)}\ \ 0\sim 50\ [\text{kWh}] \ \ 10\ [\text{yen/kWh}] \end{cases}$

Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

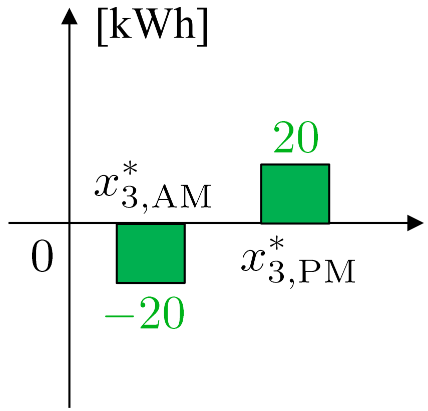
Agg 3 (batteries): $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$



Socially optimal market results **(only god knows!!)**



Agg 1



Agg 3

Optimal clearing price:

$$\begin{pmatrix} \lambda^*_{\mathrm{AM}} \\ \lambda^*_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

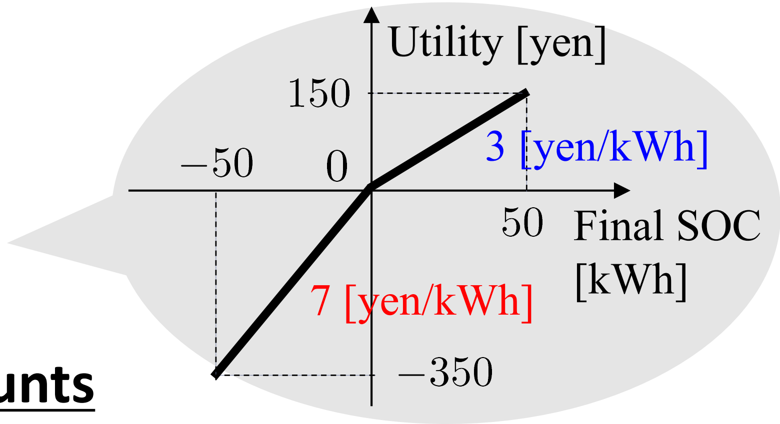
✓ optimal price levels off
(i.e. $\mu^*_{\mathrm{sft}} = 0$)

Example: Sequential Market Clearing

Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$ $\left\{ \begin{array}{lll} \text{(A)} & 0\text{~}50\ \text{[kWh]} & 5\ \text{[yen/kWh]} \\ \text{(B)} & 0\text{~}50\ \text{[kWh]} & 10\ \text{[yen/kWh]} \end{array} \right.$

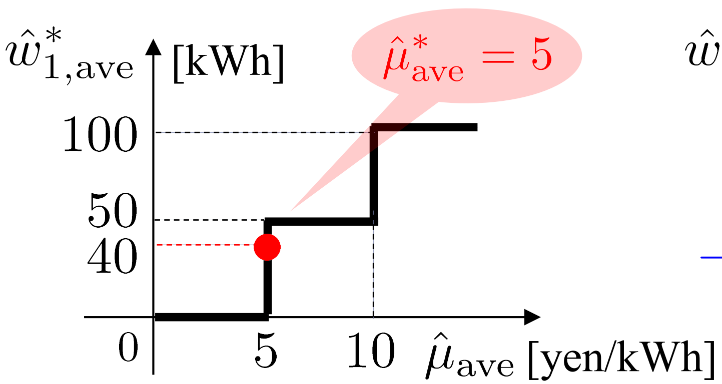
Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

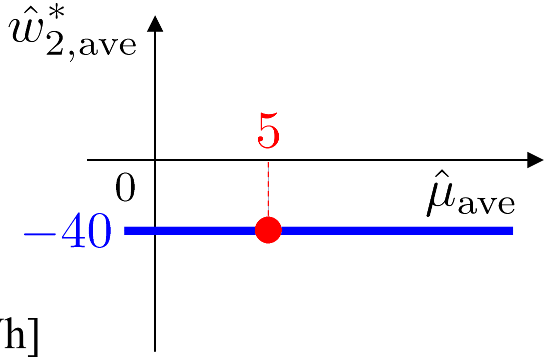
Agg 3 (batteries): $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$

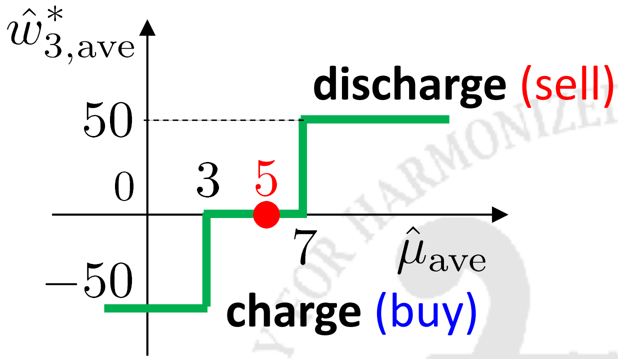


1) Market clearing of average energy amounts

Approximate bid function: $\hat{w}^*_{\alpha,\mathrm{ave}}(\hat{\mu}_{\mathrm{ave}}) = \boldsymbol{w}^*_{\alpha,\mathrm{ave}}(\hat{\mu}_{\mathrm{ave}}, \mu_{\mathrm{sft}})\big|_{\mu_{\mathrm{sft}}=0}$





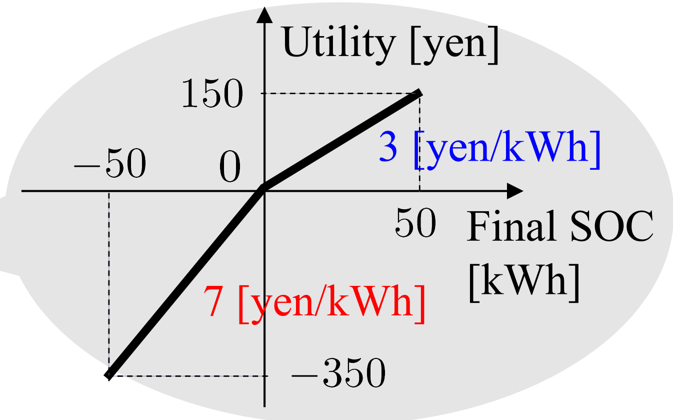


Example: Sequential Market Clearing

Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$ $\begin{cases} \text{(A)}\ \ 0\text{~}50\ [\mathrm{kWh}]\quad 5\ [\mathrm{yen/kWh}] \\ \text{(B)}\ \ 0\text{~}50\ [\mathrm{kWh}]\ \ 10\ [\mathrm{yen/kWh}] \end{cases}$

Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

Agg 3 (batteries): $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$

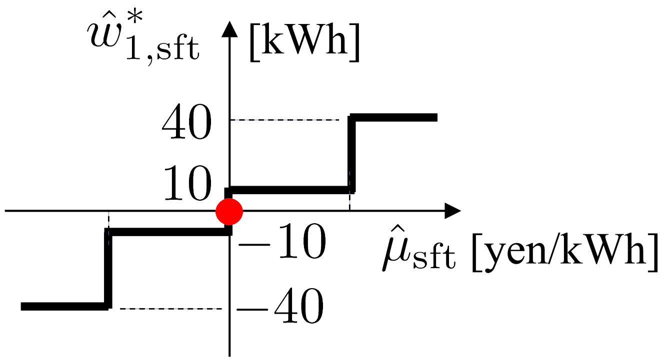


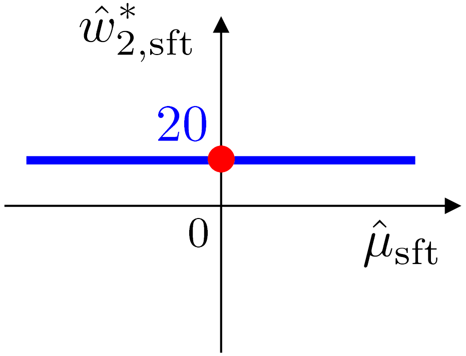
1) Market clearing of average energy amounts

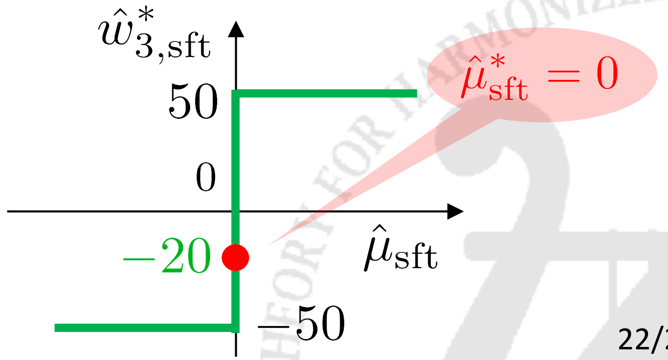
Balancing amounts: $\hat{w}^*_{1,\mathrm{ave}} = 40$ $\hat{w}^*_{2,\mathrm{ave}} = -40$ $\hat{w}^*_{3,\mathrm{ave}} = 0$ **Average price:** $\hat{\mu}^*_{\mathrm{ave}} = 5$

2) Market clearing of shift energy amounts

Approximate bid function: $\hat{w}^*_{\alpha,\mathrm{sft}}(\hat{\mu}_{\mathrm{sft}}) = \arg\max_{w_{\alpha,\mathrm{sft}}} \left\{ \hat{\mu}_{\mathrm{sft}} w_{\alpha,\mathrm{sft}} - H_\alpha(\hat{w}^*_{\alpha,\mathrm{ave}}, w_{\alpha,\mathrm{sft}}) \right\}$





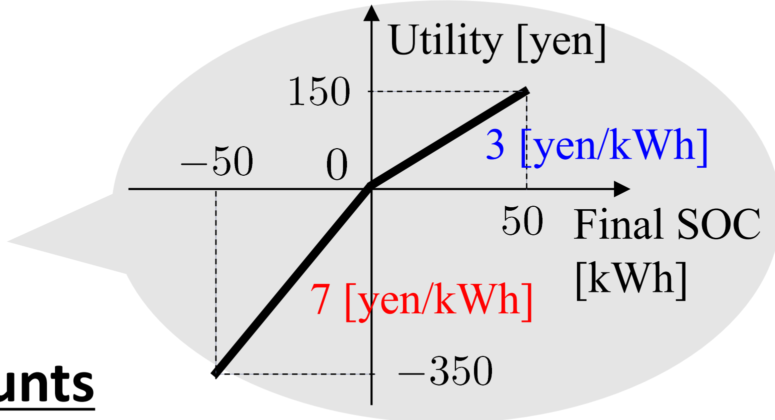


Example: Sequential Market Clearing

Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$ $\begin{cases} \text{(A)} \ \ 0\text{~}50 \text{ [kWh]} \quad 5 \text{ [yen/kWh]} \\ \text{(B)} \ \ 0\text{~}50 \text{ [kWh]} \ \ 10 \text{ [yen/kWh]} \end{cases}$

Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

Agg 3 (batteries): $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$



1) Market clearing of average energy amounts

Balancing amounts: $\hat{w}^*_{1,\mathrm{ave}} = 40$ $\hat{w}^*_{2,\mathrm{ave}} = -40$ $\hat{w}^*_{3,\mathrm{ave}} = 0$ **Average price:** $\hat{\mu}^*_{\mathrm{ave}} = 5$

2) Market clearing of shift energy amounts

Balancing amounts: $\hat{w}^*_{1,\mathrm{sft}} = 0$ $\hat{w}^*_{2,\mathrm{sft}} = 20$ $\hat{w}^*_{3,\mathrm{sft}} = -20$ **Shift energy price:** $\hat{\mu}^*_{\mathrm{sft}} = 0$

【Theorem】 Socially optimal market clearing iff optimal price levels off

i.e. $(T^{-1}\hat{w}^*_\alpha)_{\alpha \in \mathcal{A}} = (x^*_\alpha)_{\alpha \in \mathcal{A}}, \ T^{-1}\hat{\mu}^* = \lambda^* \iff \lambda^*_1 = \cdots = \lambda^*_n.$

Example: Sequential Market Clearing

Agg 1 (generators): $\begin{pmatrix} x_{1,\mathrm{AM}} \\ x_{1,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} g_{\mathrm{AM}} \\ g_{\mathrm{PM}} \end{pmatrix}$ $\begin{cases} \text{(A)}\ \ 0\sim 50\ [\text{kWh}]\ \ \ 5\ [\text{yen/kWh}] \\ \text{(B)}\ \ 0\sim 50\ [\text{kWh}]\ \ 10\ [\text{yen/kWh}] \end{cases}$

Agg 2 (loads): $\begin{pmatrix} x_{2,\mathrm{AM}} \\ x_{2,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -l_{\mathrm{AM}} \\ -l_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} -20 \\ -60 \end{pmatrix}$

Agg 3 (batteries): **(without battery aggregator)** $\begin{pmatrix} x_{3,\mathrm{AM}} \\ x_{3,\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} \delta^{\mathrm{out}}_{\mathrm{AM}} - \delta^{\mathrm{in}}_{\mathrm{AM}} \\ \delta^{\mathrm{out}}_{\mathrm{PM}} - \delta^{\mathrm{in}}_{\mathrm{PM}} \end{pmatrix}$

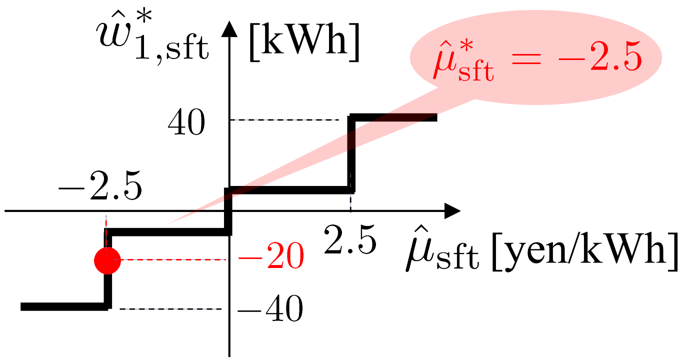
Optimal clearing price:

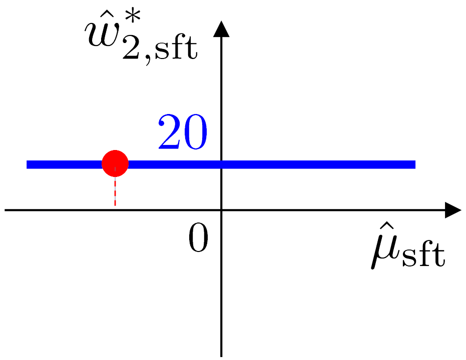
$$\begin{pmatrix} \lambda^*_{\mathrm{AM}} \\ \lambda^*_{\mathrm{PM}} \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

1) Market clearing of average energy amounts

Balancing amounts: $\hat{w}^*_{1,\mathrm{ave}} = 40$ $\hat{w}^*_{2,\mathrm{ave}} = -40$ $\hat{w}^*_{3,\mathrm{ave}}$ **(w/o)** $= 0$ **Average price:** $\hat{\mu}^*_{\mathrm{ave}} = 5$

2) Market clearing of shift energy amounts





Clearing price:

$$\begin{pmatrix} 2.5 \\ 7.5 \end{pmatrix} = \hat{\mu}^*_{\mathrm{ave}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \hat{\mu}^*_{\mathrm{sft}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\neq \begin{pmatrix} \lambda^*_{\mathrm{AM}} \\ \lambda^*_{\mathrm{PM}} \end{pmatrix}$$

At least approximate clearing even if optimal price does not level off

Numerical Example

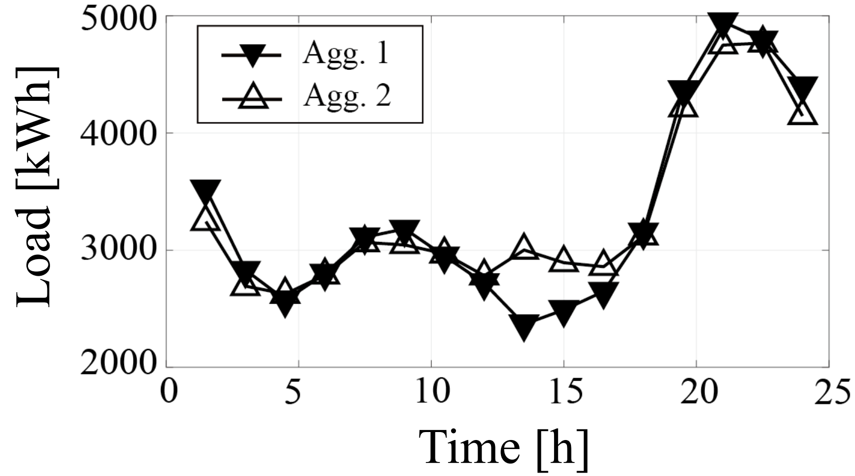
Agg 1, Agg 2 (loads & batteries)

Charge/discharge efficiencies: $\begin{cases} 95\% \text{ (Agg 1)} \\ 94\% \text{ (Agg 2)} \end{cases}$

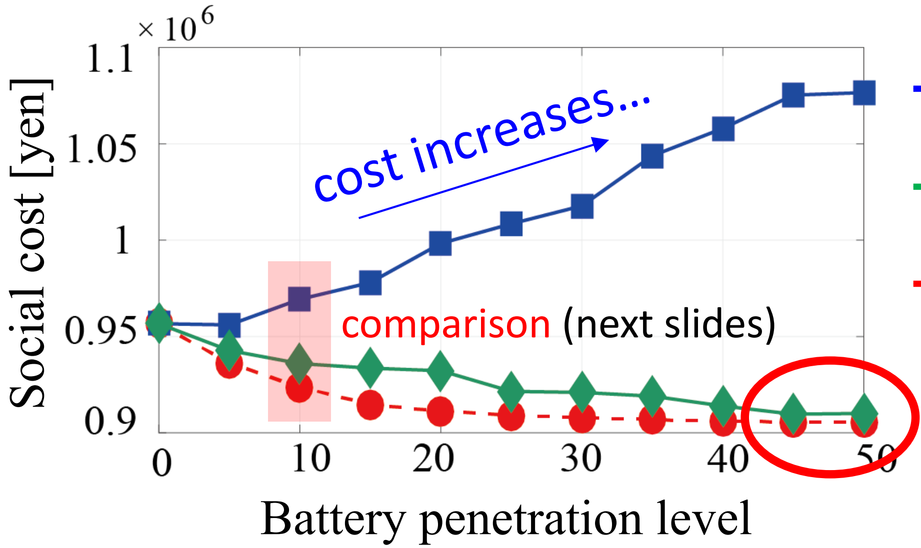
SOC/inverter constraints: **parameters**

Agg 3 (9 types of generators)

Generation costs: 3, 6,…, 27 [yen/kWh]



<u>Resultant social costs when varying battery penetration levels</u> (16 time spots)



: time domain (sequential clearing)

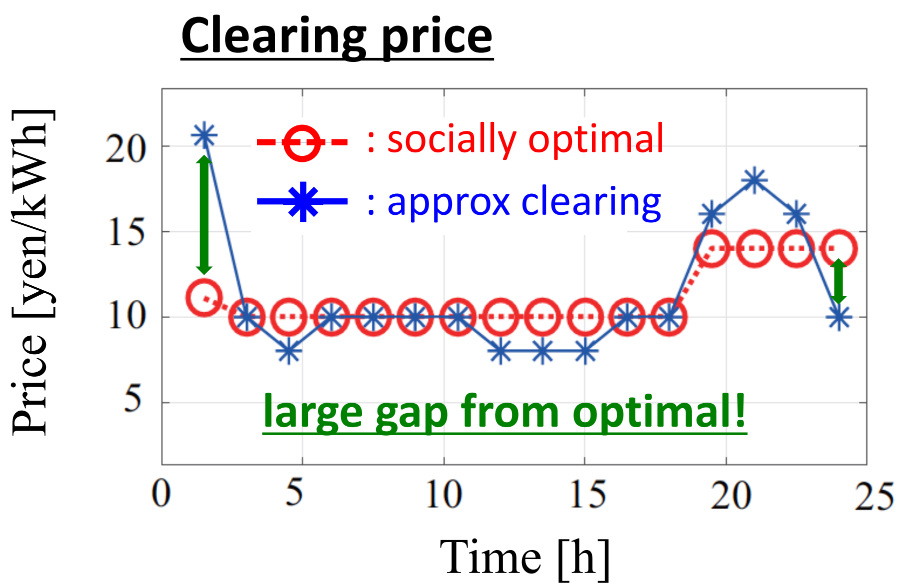
: Fourier-like domain (sequential clearing)

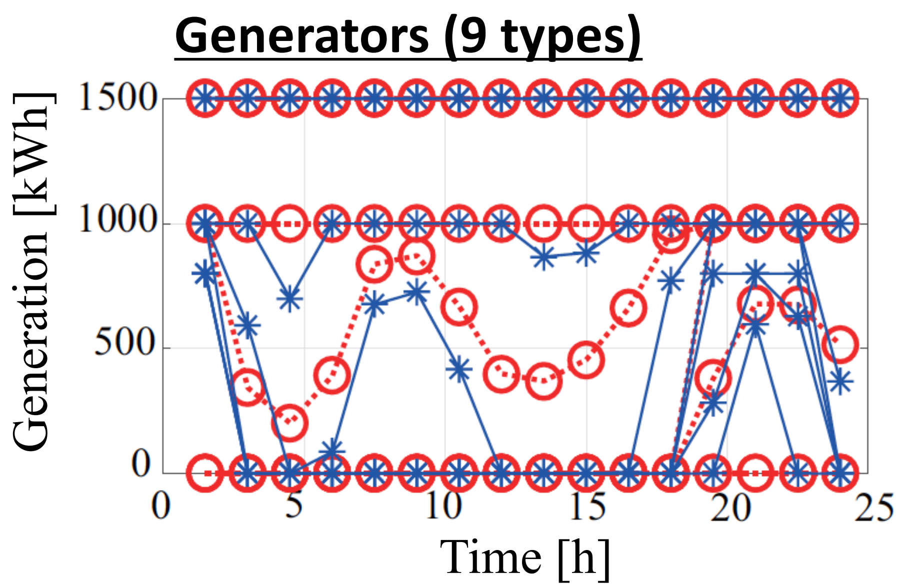
: socially optimal (**only god knows**)

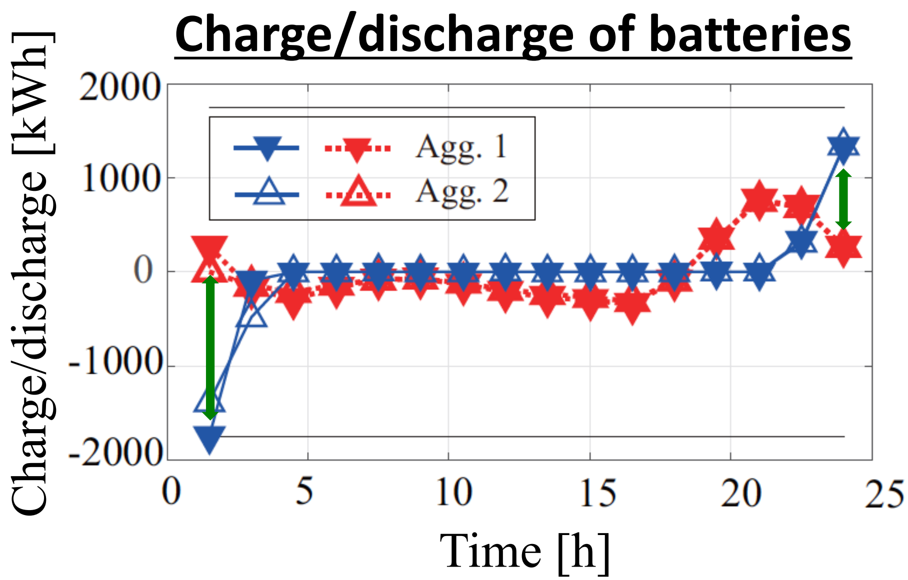
<u>Almost optimal market clearing</u>
<u>when **high battery penetration!**</u>

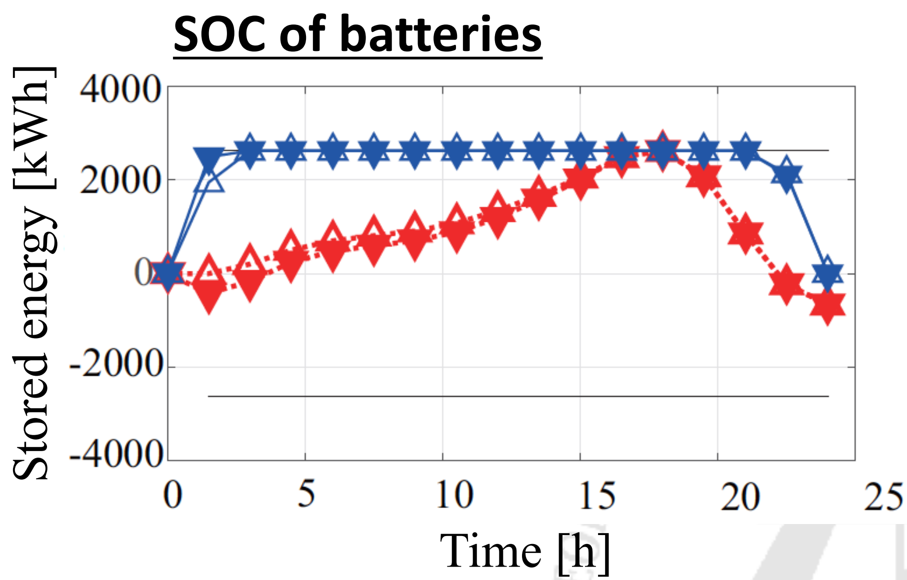
→ **price levelling-off**

Sequential Clearing in Time Domain



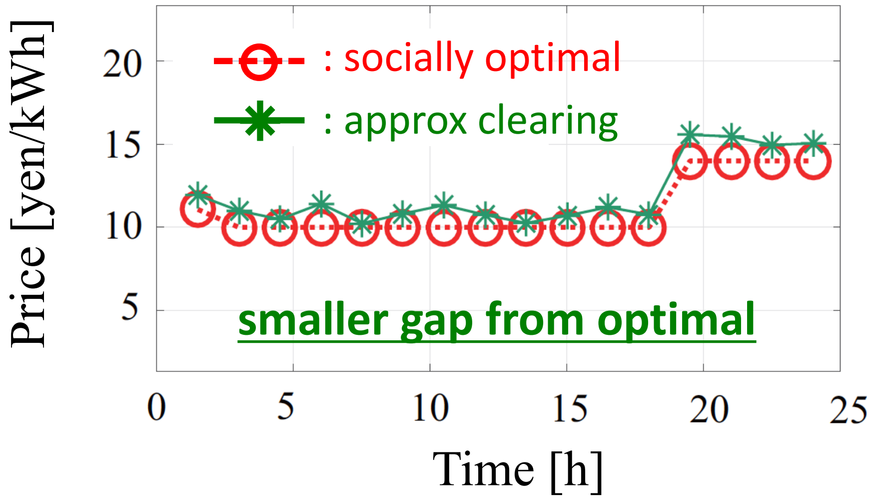




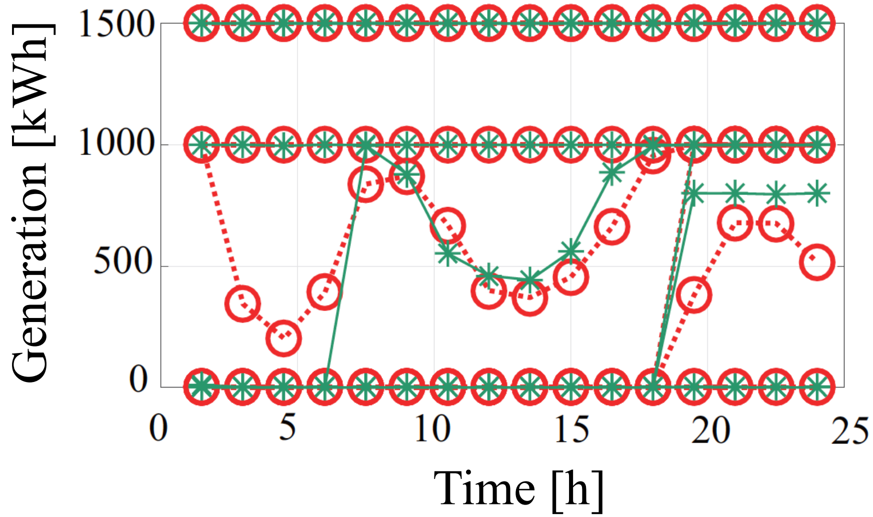


Sequential Clearing in Fourier-Like Domain

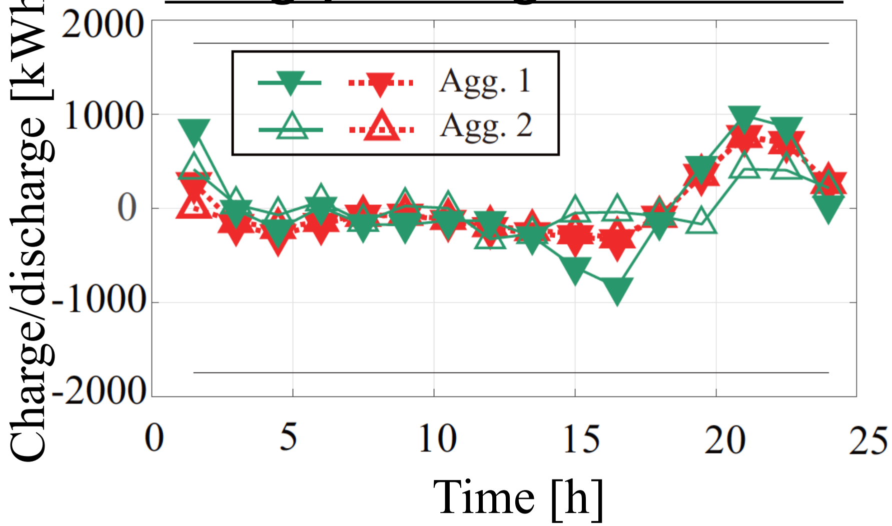
Clearing price



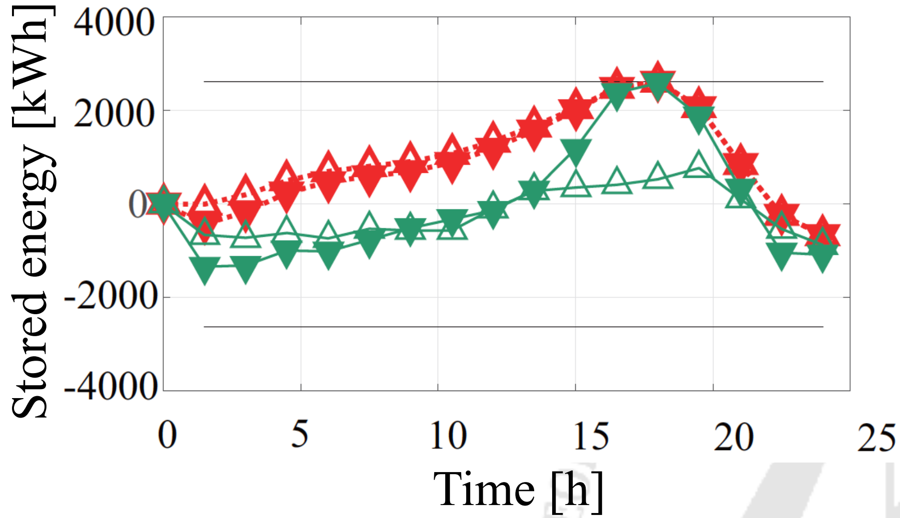
Generators (9 types)



Charge/discharge of batteries



SOC of batteries



Concluding Remarks

- **Bidding system design for multiperiod electricity markets**
  - **Distributed algorithm design for convex optimization**
    - Each aggregator submits bidding curves to ISO
    - ISO finds clearing price and balancing amounts by bidding curves
- **Proposed approach to bidding system design**
  - Basis transformation compatible with energy shift markets
  - Sequential clearing scheme based on approximate bidding curves

A Distributed Scheme for Power Profile Market Clearing under High Battery Penetration, IFAC WC 2017

Bidding System Design for Multiperiod Electricity Markets: Pricing of Stored Energy Shiftability, CDC 2017 (to appear)

Thank you for your attention!