The 4th Multi-Symposium on Control Systems

A Compensation Principle for Controller Retrofit

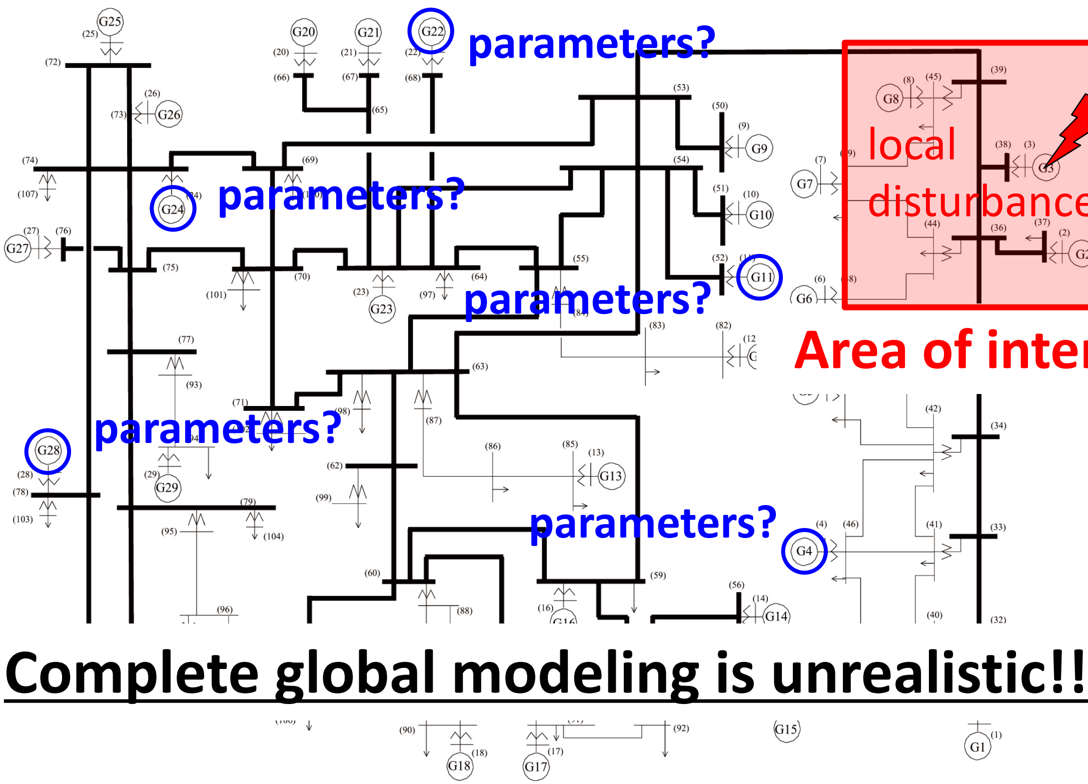
Takayuki Ishizaki (Tokyo Institute of Technology)

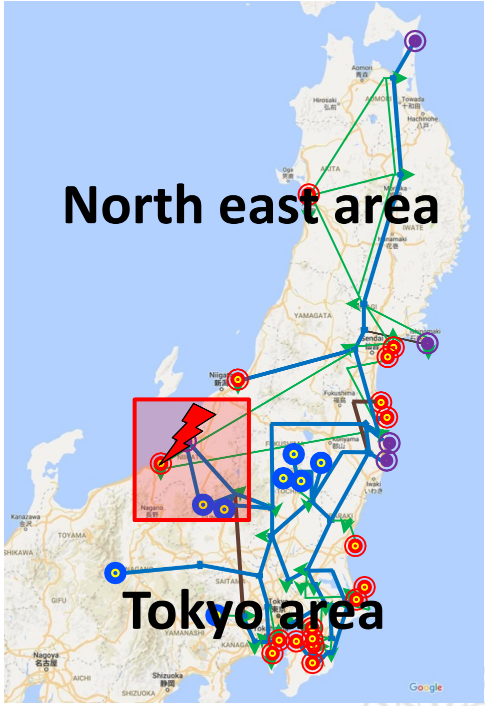
Tomonori Sadamoto, Jun-ichi Imura (Tokyo Institute of Technology)

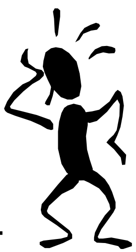
✓ **Retrofit** = add new parts or substitute modernized equipment for preexisting ones

Motivation from Power Systems Control

IEEJ EAST30 Model (stable system composed of 30 generators)



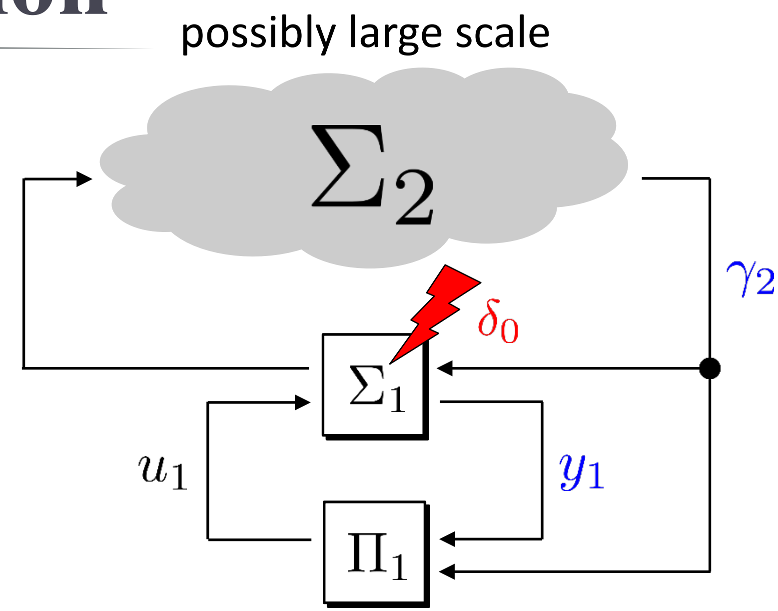




Complete global modeling is unrealistic!!

Controller design by local model?
Stability? Better performance?

Problem Formulation



Subsystem of interest (model available)

$$\Sigma_1 : \begin{cases} \dot{x}_1 = \boldsymbol{A}_1 x_1 + \boldsymbol{L}_1 \gamma_2 + \boldsymbol{B}_1 u_1 \\ y_1 = \boldsymbol{C}_1 x_1 \end{cases}$$

Other subsystem(s) (model unavailable)

$$\Sigma_2 : \begin{cases} \dot{x}_2 = A_2 x_2 + L_2 \varGamma_1 x_1 \\ \gamma_2 = \varGamma_2 x_2 \end{cases}$$

✓ $x_1(0) = \delta_0,\ x_2(0) = 0,\ \|\delta_0\| \leq 1$

Assumption:
(i) y_1, γ_2 are measurable
(ii) the preexisting system without Π_1 is stable

【Problem】 Find a **retrofit controller** $\Pi_1 : u_1 = \mathcal{K}_1(y_1, \gamma_2)$ such that
(a) the whole system is **kept stable** and (b) $\|x_1\|_{\mathcal{L}_2}$ **is made small** for any δ_0.

Hierarchical State-Space Expansion

Coupled state equation of Σ_1 and Σ_2

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{L}_1 \Gamma_2 \\ L_2 \Gamma_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1 \\ 0 \end{bmatrix} u_1$$

stable (assumption)

state-space expansion to **cascade realization**

Hierarchical realization $(2n_1 + n_2)$-dim

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{L}_1 \Gamma_2 \\ L_2 \Gamma_1 & A_2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2 \Gamma_1 \end{bmatrix} \hat{\xi}_1$$

stable

decoupled!

$$\dot{\hat{\xi}}_1 = \boldsymbol{A}_1 \hat{\xi}_1 + \boldsymbol{B}_1 u_1$$

stabilized by u_1

【Lemma】 If $\xi_1(0) = 0,\ \xi_2(0) = 0$ and $\hat{\xi}_1(0) = \delta_0$

then $x_1(t) \equiv \xi_1(t) + \hat{\xi}_1(t)$ and $x_2(t) \equiv \xi_2(t)$ for any $u_1(t)$.

Localized Controller Design

Hierarchical realization

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{L}_1 \Gamma_2 \\ L_2 \Gamma_1 & A_2 \end{bmatrix}}_{\text{stable}} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2 \Gamma_1 \end{bmatrix} \hat{\xi}_1$$

model available!

$$\dot{\hat{\xi}}_1 = \boldsymbol{A}_1 \hat{\xi}_1 + \boldsymbol{B}_1 u_1$$

【Lemma】 Design a controller $u_1 = \boldsymbol{K}_1 \boldsymbol{C}_1 \hat{\xi}_1$ such that

$$\dot{\hat{\xi}}_1 = (\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{K}_1 \boldsymbol{C}_1)\hat{\xi}_1 \text{ is stable and } \|\hat{\xi}_1\|_{\mathcal{L}_2} \leq \mu_1.$$

Then the closed-loop system is stable and $\|\xi_1 + \hat{\xi}_1\|_{\mathcal{L}_2} \leq \alpha_1 \mu_1, \ \forall \delta_0.$

constant

✓ Generalization to dynamical controller design is straightforward

How to implement $u_1 = \boldsymbol{K}_1 \boldsymbol{C}_1 \hat{\xi}_1$ **??**

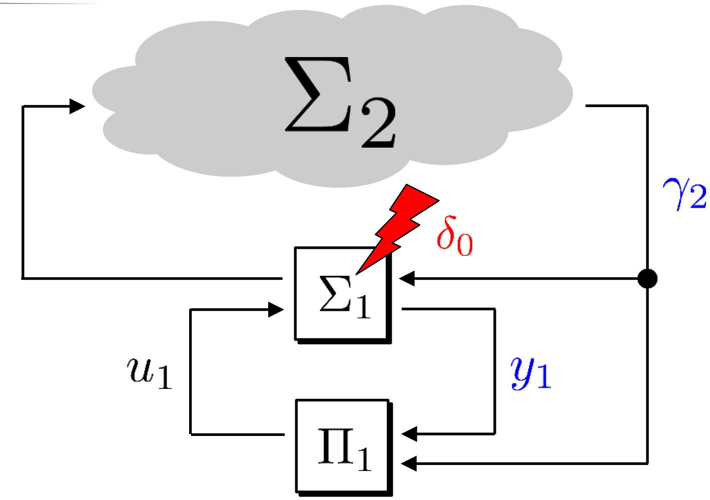
$\neq y_1$

Controller Implementation

How to implement $u_1 = \boldsymbol{K}_1 \boldsymbol{C}_1 \hat{\xi}_1$ **??**

$$\boldsymbol{C}_1 \hat{\xi}_1(t) \equiv \boldsymbol{C}_1 x_1(t) - \boldsymbol{C}_1 \xi_1(t)$$

$$\Gamma_2 \xi_2(t) \equiv \Gamma_2 x_2(t) \equiv \gamma_2(t)$$



$\dot{\xi}_1 = \boldsymbol{A}_1 \xi_1 + \boldsymbol{L}_1 \Gamma_2 \xi_2$ with $\xi_1(0) = 0$

$\hat{x}_1(t) \equiv \xi_1(t)$

$\Longleftrightarrow$ $\dot{\hat{x}}_1 = \boldsymbol{A}_1 \hat{x}_1 + \boldsymbol{L}_1 \gamma_2$ with $\hat{x}_1(0) = 0$

【Theorem】 The closed-loop system with the retrofit controller

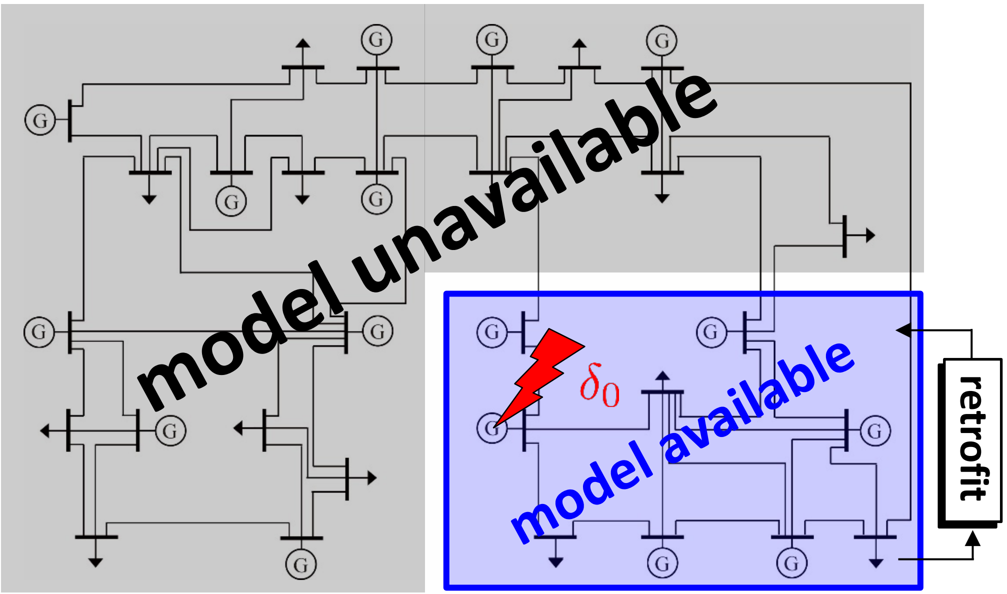
$$\Pi_1 : \begin{cases} \dot{\hat{x}}_1 = \boldsymbol{A}_1 \hat{x}_1 + \boldsymbol{L}_1 \gamma_2 \\ u_1 = \boldsymbol{K}_1 (y_1 - \boldsymbol{C}_1 \hat{x}_1) \end{cases}$$

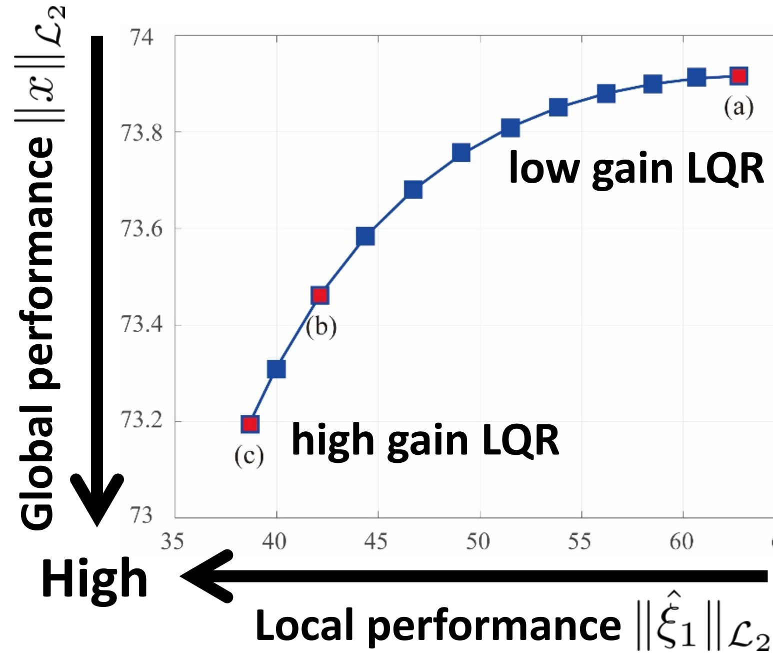
Compensator

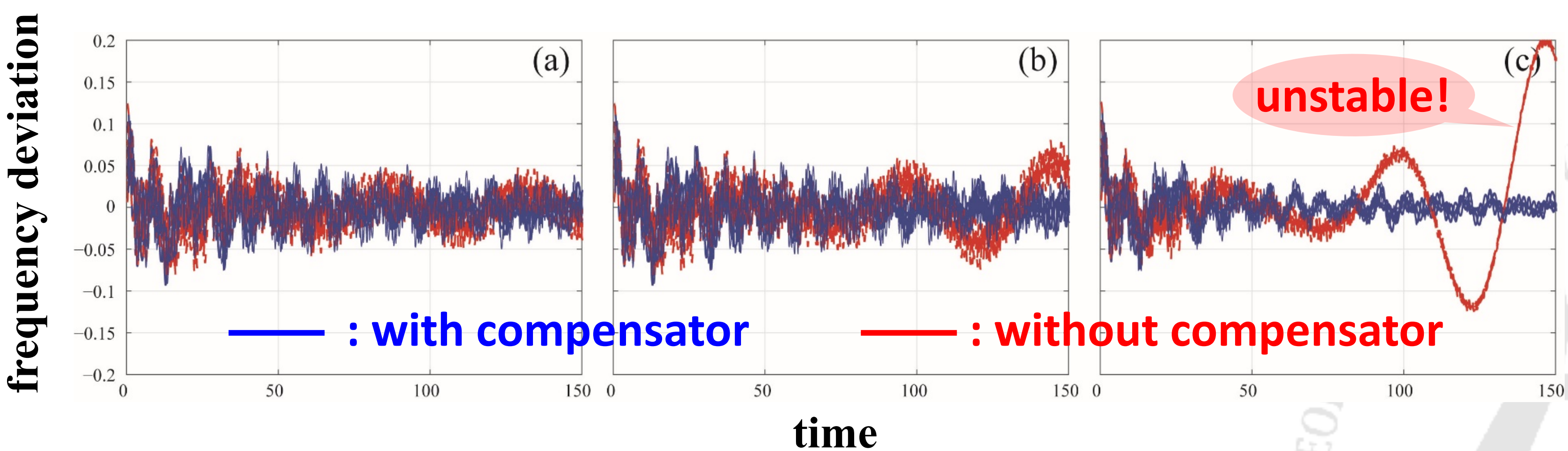
is internally stable and it satisfies $\|x_1\|_{\mathcal{L}_2} \le \alpha_1 \mu_1, \ \forall \delta_0$.

by T. Sadamoto (Tokyo Inst. of Tech.)

Application to Power Systems Control







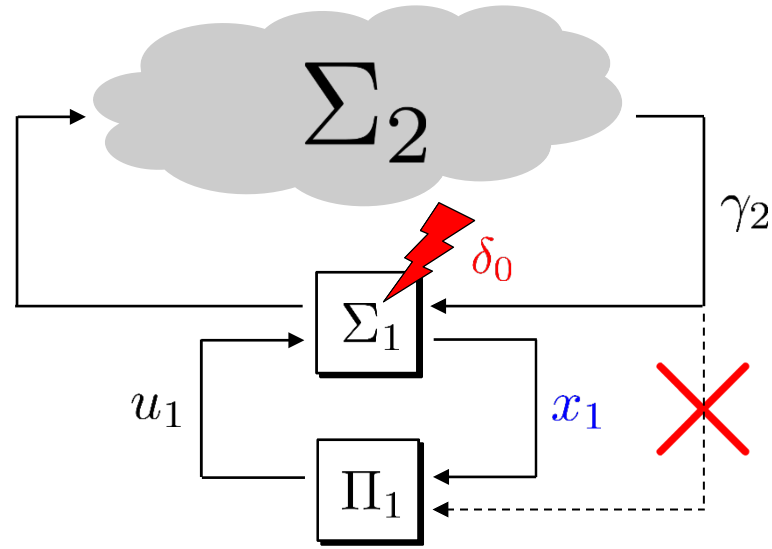
Retrofit Control without Interconnection Signal Measurement?

Subsystem of interest (model available)

$$\Sigma_1 : \begin{cases} \dot{x}_1 = \boldsymbol{A}_1 x_1 + \boldsymbol{L}_1 \gamma_2 + \boldsymbol{B}_1 u_1 \\ y_1 = x_1 \end{cases}$$

Other subsystem(s) (model unavailable)

$$\Sigma_2 : \begin{cases} \dot{x}_2 = A_2 x_2 + L_2 \Gamma_1 x_1 \\ \gamma_2 = \Gamma_2 x_2 \end{cases}$$



Assumption: (i) x_1 is measurable
(ii) the preexisting system without Π_1 is stable

【Problem】 Find a retrofit controller $\Pi_1 : u_1 = \mathcal{K}_1(x_1)$ such that
(a) the whole system is **kept stable** and (b) $\|x_1\|_{\mathcal{L}_2}$ **is made small** for any δ_0.

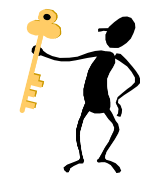
Parameterization via Projection

Coupled state equation of Σ_1 and Σ_2

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{L}_1\Gamma_2 \\ L_2\Gamma_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1 \\ 0 \end{bmatrix} u_1$$

stable

Introduction of **projector** $\boldsymbol{P}_1 \in \mathbb{R}^{n \times \hat{n}}$



✓ $\boldsymbol{P}_1 \boldsymbol{P}_1^{\dagger} + \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} = I$

Hierarchical realization $(n_1 + n_2 + \hat{n}_1)$-dim

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{L}_1\Gamma_2 \\ L_2\Gamma_1 & A_2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} \boldsymbol{A}_1 \\ L_2\Gamma_1 \end{bmatrix} \hat{\xi}_1$$

stable

stabilized by u_1

$$\dot{\hat{\xi}}_1 = \boldsymbol{P}_1^{\dagger} \boldsymbol{A}_1 \boldsymbol{P}_1 \hat{\xi}_1 + \boldsymbol{P}_1^{\dagger} \boldsymbol{B}_1 u_1$$

【Lemma】 If $\xi_1(0) = \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} \delta_0,\ \xi_2(0) = 0,\ \hat{\xi}_1(0) = \boldsymbol{P}_1^{\dagger} \delta_0$ and $\mathrm{im}\, \boldsymbol{B}_1 \subseteq \mathrm{im}\, \boldsymbol{P}_1$,

then $x_1(t) \equiv \xi_1(t) + \boldsymbol{P}_1 \hat{\xi}_1(t)$ and $x_2(t) \equiv \xi_2(t)$ for any $u_1(t)$.

Controller $u_1 = \hat{\boldsymbol{K}}_1 \hat{\xi}_1$ such that $\|\hat{\xi}_1\|_{\mathcal{L}_2} \leq \mu_1$ **How to implement??**

Retrofit Controller Implementation

Controller & Compensator

$$u_1 = \hat{\boldsymbol{K}}_1 \hat{\xi}_1 = \hat{\boldsymbol{K}}_1 (\boldsymbol{P}_1^{\dagger} x_1 - \boldsymbol{P}_1^{\dagger} \xi_1)$$

$\hat{x}_1(t) = \boldsymbol{P}_1^{\dagger} \xi_1(t)$

$$\dot{\hat{x}}_1 = \boldsymbol{P}_1^{\dagger} \boldsymbol{A}_1 \boldsymbol{P}_1 \hat{x}_1 + \boldsymbol{P}_1^{\dagger} \boldsymbol{A}_1 \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} x_1 + \boldsymbol{P}_1^{\dagger} \boldsymbol{L}_1 \Gamma_2 x_2 \quad \text{with} \quad \hat{x}_1(0) = 0$$

unavailable

【**Lemma**】 If $\operatorname{rank} \boldsymbol{B}_1 + \operatorname{rank} \boldsymbol{L}_1 \leq n_1$, $\operatorname{im} \boldsymbol{B}_1 \cap \operatorname{im} \boldsymbol{L}_1 = \emptyset$, then there exist $\boldsymbol{P}_1$ and $\boldsymbol{P}_1^{\dagger}$ such that $\operatorname{im} \boldsymbol{B}_1 \subseteq \operatorname{im} \boldsymbol{P}_1$, $\operatorname{im} \boldsymbol{L}_1 \subseteq \ker \boldsymbol{P}_1^{\dagger}$.

【**Theorem**】 The closed-loop system with the retrofit controller

$$\Pi_1 : \begin{cases} \dot{\hat{x}}_1 = \boldsymbol{P}_1^{\dagger} \boldsymbol{A}_1 \boldsymbol{P}_1 \hat{x}_1 + \boldsymbol{P}_1^{\dagger} \boldsymbol{A}_1 \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} x_1 \\ u_1 = \hat{\boldsymbol{K}}_1 (\boldsymbol{P}_1^{\dagger} x_1 - \hat{x}_1) \end{cases}$$

due to $\overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^{\dagger} \delta_0$ uncontrollable

is internally stable and it satisfies $\|x_1\|_{\mathcal{L}_2} \leq \alpha_1 \mu_1 + \beta_1, \ \forall \delta_0.$

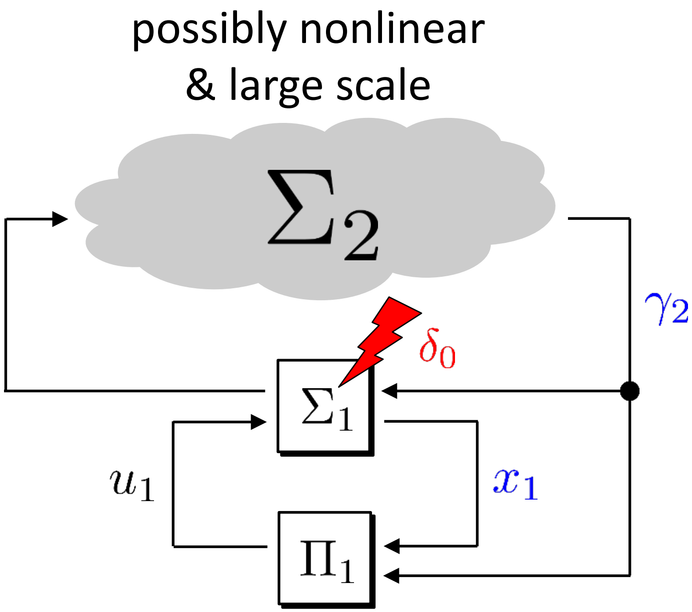
Generalization to Nonlinear Systems

Subsystem of interest (model available)

$$\Sigma_1 : \begin{cases} \dot{x}_1 = \boldsymbol{A}_1 x_1 + \boldsymbol{f}_1(x_1) + \boldsymbol{L}_1 \gamma_2 + \boldsymbol{B}_1 u_1 \\ y_1 = x_1 \end{cases}$$

Other subsystem(s) (model unavailable)

$$\Sigma_2 : \begin{cases} \dot{x}_2 = f_2(x_2, x_1) \\ \gamma_2 = h_2(x_2, x_1) \end{cases}$$

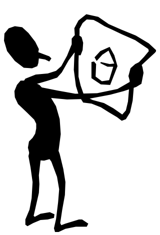


Parameterized retrofit controller (state feedback)

$$\Pi_1 : \begin{cases} \dot{\hat{x}}_1 = \boldsymbol{P}_1^\dagger \boldsymbol{A}_1 \boldsymbol{P}_1 \hat{x}_1 + \boldsymbol{P}_1^\dagger \boldsymbol{f}_1(x_1) + \boldsymbol{P}_1^\dagger \boldsymbol{A}_1 \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^\dagger x_1 + \boldsymbol{P}_1^\dagger \boldsymbol{L}_1 \gamma_2 \\ u_1 = \hat{\boldsymbol{K}}_1 (\boldsymbol{P}_1^\dagger x_1 - \hat{x}_1) \end{cases}$$

vanishes if $\boldsymbol{P}_1 = I$ (for $\boldsymbol{P}_1^\dagger \boldsymbol{A}_1 \overline{\boldsymbol{P}}_1 \overline{\boldsymbol{P}}_1^\dagger x_1$)

vanishes if $\mathrm{im}\, \boldsymbol{L}_1 \subseteq \ker \boldsymbol{P}_1^\dagger$ (for $\boldsymbol{P}_1^\dagger \boldsymbol{L}_1 \gamma_2$)



Controller design is based on local linear dynamics:

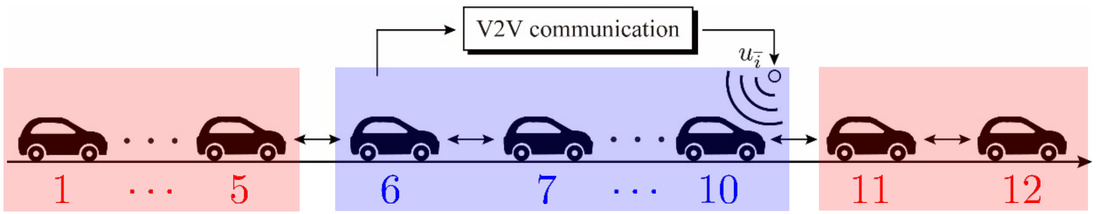
Design $u_1 = \hat{\boldsymbol{K}}_1 \hat{\xi}_1$ making $\|\hat{\xi}_1\|_{\mathcal{L}_2}$ small for $\dot{\hat{\xi}}_1 = \boldsymbol{P}_1^\dagger \boldsymbol{A}_1 \boldsymbol{P}_1 \hat{\xi}_1 + \boldsymbol{P}_1^\dagger \boldsymbol{B}_1 u_1$

Application to Collision Avoidance Control

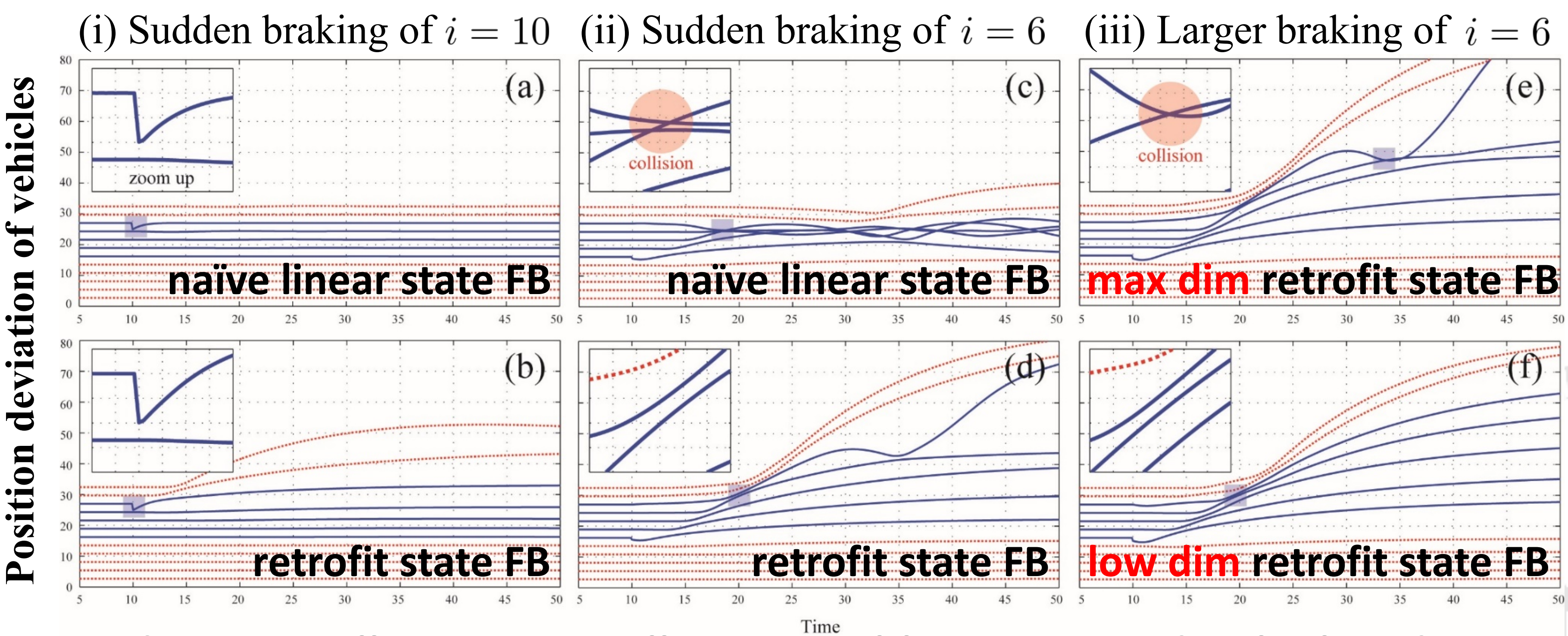
by T. Sadamoto (Tokyo Inst. of Tech.)

driver property to avoid collision

Vehicle dynamics $\dot{p}_i = v_i, \quad \dot{v}_i = \kappa \{ a(p_{i+1} - p_i) b(p_i - p_{i-1}) - v_i \} + u_i$



- **Subsystem of interest**
- **Other subsystems**



Low dim controller is practically reasonable as opposed to higher dim one

Concluding Remarks

- **Retrofit control**
  - Localization of controller design and implementation
  - Stability guarantee and control performance improvement
- **Hierarchical state-space expansion**
  - Redundant realization with cascade structure
  - Systematic analyses for stability and control performance

Systematic controller retrofit is enabled by a compensator that cancels out interference due to the dynamics neglected in localized controller design

Thank you for your attention!